Proving Social Choice Possible

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Abstract

In 1951 Kenneth Arrow published a book in which he proved that social choice was impossible. There was no way to amalgamate individual preferences into a social preference in such a way that certain rational and normative conditions were met. Later Gibbard and Satterthwaite proved that any such amalgamation of individual preferences in which there was no advantage to any individual to use strategy to order their preferences insincerely in order to get a better result for themselves was also impossible. These impossibility theorems have been thought to rule out direct democracy and also welfare economics giving credibility to the implication that representative democracy and capitalist economics are the best systems that can be devised.

Instead of simple amalgamation, we have devised a more general information processing system which represents the implementation of a mechanism that accepts inputs from individual choosers as utilitarian ratings and outputs a social choice in the form of a complete ordinal ranking. It is a hybrid utilitarian approval system (UAV). This system is designed to disincentivize individual choosers from choosing strategically or insincerely. The system itself maximizes the efficacy of each individual input. The information is processed in such a way as to alleviate concerns about interpersonal comparisons of utility. A theorem is proved that this mechanism satisfies Arrow's five rational and normative conditions, and, because of the more finely tuned normative data, it is even more robust normatively. This mechanism produces the utilitarian

winner(s), the one(s) which maximizes social utility, and a maximin condition can be implemented. The result is that a utility based social choice system has been devised which overcomes both impossibility theorems and should give new life to welfare economics and direct democracy.

Introduction

In Social Choice and Individual Values, Kenneth Arrow (1951: p. 1) wrote "In a capitalist democracy there are essentially two methods by which social choices can be made: voting, typically used to make 'political' decisions, and the market mechanism, typically used to make 'economic' decisions." Initially, Arrow does not distinguish between political and economic systems claiming that both are means of formulating social decisions based on individual inputs. Arrow then purports to show that there is no way to make social decisions based on the amalgamation of individual ones, assuming certain rational and normative conditions are met, thus ruling out welfare economics, economic democracy and direct political democracy. The dichotomy between political and economic systems remains with the implication that representative democracy and capitalist economics are the best systems that can be devised. Arrow's result, formerly called the *paradox of voting,* was first discovered by the Marquis de Condorcet (1785). Condorcet's paradox showed that majority preferences can become intransitive when there are three or more alternatives. Arrow basically mathematized Condorcet's insight.

Gibbard and Satterthwaite concurred with Arrow and proved that any social choice system that was strategy proof was also impossible. Gibbard (1973: p. 587) states: "An individual 'manipulates' the choosing scheme if, by misrepresenting his preferences, he secures an outcome he prefers to the 'honest' outcome - the choice the community would make if he expressed his true preferences." Satterthwaite (1975: p.188) showed that the requirement for voting procedures of strategyproofness and Arrow's requirements for social welfare functions are equivalent: "a one-to-one correspondence exists between every strategy-proof voting procedure and every social welfare function satisfying Arrow's four requirements." Jackson (2001: p. 2) states: "Often, one thinks of the desired outcomes as the given and analyzes whether there exist game forms for which the strategic properties induce individuals to (always) choose actions that lead to the desired outcomes." We design a game form for which the strategic properties induce individuals to choose actions that lead to the desired outcome – a possible social choice – while disincentivizing them from choosing strategically as individuals. We show that this mechanism also satisfies Arrow's rational and normative conditions.

Gibbard's results were based only on the possibility that someone could use strategy if they were astute enough to stumble on a way to do so. (1973: p. 590) "Note that to call a voting scheme manipulable is not to say that, given the actual circumstances, someone is really in a position to manipulate it." Only the possibility exists in an elaborate mathematical structure. Gibbard doesn't assume that there is any formularizable or identifiable strategy that an individual chooser could use to manipulate the system. Other writers have pointed out this difficulty: (Meir et. al.: p. 149) "In other words, computational complexity may be an obstacle that prevents strategic behavior." By contrast, we analyze a situation in which an actual identifiable strategy exists which can be known both to the individual chooser and to the mechanism, which amalgamates or processes the choices, itself. If the mechanism does the strategizing for each individual, there is no incentive for the individual to do so.

Gibbard's and Satterthwaite's analysis is deterministic while the problem of manipulability is inherently probabilistic. In an actual election it would be impossible for a voter to know the ideal strategy unless they knew how every other voter was going to vote. Polling, however, can provide some information of a probabilistic nature about other voters. We incorporate the fundamentally probabilistic nature of the choosing process in our analysis, and the mechanism we develop is generalizable to the situation in which polling data is available. Other writers (Cranor,1996; LeGrand, 2008) have also sought to develop systems such as Declared-Strategy Voting which attempts to "elicit more sincere preferences from voters ... to find a winning alternative in such a way that voters would be unlikely to gain a superior result by submitting insincere preferences."

Aki Lehtinen (2011: p.376) concludes that Arrow's Impossibility Theorem is not relevant in the final analysis:

> “Arrow’s impossibility result and the closely related theorems given by Gibbard and Satterthwaite are unassailable as deductive proofs. However, we should not be concerned about these results because their most crucial conditions are not justifiable. Fortunately, we know that strategy-proofness is usually violated under

> all voting rules and that IIA [Independence of Irrelevant Alternatives] does not preclude strategic voting."

Unlike Lehtinen, we do not dispute the Arrow and Gibbard-Satterthaite analyses and conclusions. Their mathematics is impeccable. Instead, by thinking outside the box, we analyze a social choice mechanism which accomplishes what Arrow, Gibbard and Satterthwaite purportedly set out to accomplish – a system that produces a social choice based on individual inputs and which exemplifies certain rational and normative criteria including strategyproofness. The mechanism analyzed here accomplishes this in a manner that not only is more realistically implementable in terms of actual voting/choosing systems but is also more robust normatively.

A major stumbling block for the development of utilitarian social choice systems regards the issue of interpersonal comparisons. It has been thought that scales which measure the utilities of individuals are incompatible, and that any scale chosen, upon which all individuals are supposed to rate their utilities, would be arbitrary. Arrow (1951: p. 9) states: "The viewpoint will be taken here that interpersonal comparison of utilities has no meaning and, in fact, that there is no meaning relevant to welfare comparisons in the measurability of individual utility." Thus, according to Arrow, any individual input must be based on individual preference rankings of the form aRbRc..., meaning a is preferred or indifferent to b, b is preferred or indifferent to c etc. Although "comparisons in the measurability of individual utilities" may have no meaning when done by an outside observer, the assertion of utilities by individuals themselves on a scale of their own choosing certainly does. Furthermore, basing all inputs on the form aRbRc tacitly assumes that there is an equality of utility scales among all inputs.

We assume that choosers can place their respective utilities for alternatives on a scale of their own choosing on a line consisting of the set of all non-negative real numbers, $\mathbb{R}_{\geq 0}$, and also choose the end points. In general there will be a utility for each possible alternative specified by each chooser. We will show that, for the mechanism modeled here, any affine linear transformation of an individual's set of utility ratings will yield the same social choice result, and, therefore, it doesn't matter which scale an individual chooses. This is not to say that the utility scale chosen by an individual is not meaningful to the individual themselves, but just that, whatever scale they choose, their contribution to the final output of the mechanism we analyze will be the same. Any affine linear transformation of a chooser's utility scale will yield the same results since n^*_j will be the same before and after the transformation.

Sen's (1970) Cardinal Non-Comparability condition(CNC) states that for all $U_j, U'_j \in \mathbb{R}$, one has $f(U_j) = f(U'_j)$ whenever for all $j \in V$, there are real constants κ_j and ν_j, with each $\nu_j > 0$, such that $U'_j \equiv \kappa_j + \nu_j U_j$. The social choice mechanism detailed in this paper is invariant to affine rescaling of utilities since the optimal threshold is a function of $n^*_j\ \forall j$.

There is a transformation from cardinal information to ordinal information since each utility profile, **u** = $(u_1, u_2, ..., u_n)$ is converted to a vector composed of integers, **v** = $(v_1, v_2, ..., v_n) \in \{0,1\}$. So even though the individual utilities are cardinal noncomparable, the transformed utilities are ordinal and comparable by assumption. What's more, the individual utilities and social utility of the final results are computable since the original individual utilities are known to the system. A maximin or leximin transformation of the results is also possible making cardinal full comparability unnecessary. Since we prove later that the OTM mechanism results in the social choice which maximizes social utility, (the utilitarian winner), implementing a maximin or leximin condition diminishes the utility of the social choice in order to insure that each participant has at least a minimum utility in the social choice. Contrary to Arrow's assertion that the interpersonal comparison of utilities has no meaning, we assert that making the scales of all personal utilities the same at the input and adjusting the social choice so that everyone has at least a minimum of individual utility (a maximin confition) is meaningful.

We develop a social choice mechanism that is utility based, but which overcomes the objections of arbitrariness of individual utility scales, is strategyproof and also meets an upgraded version of Arrow's normative and rational criteria.

Utilitarian and Approval Choosing

Utilitarian and approval choosing are exactly analogous to utilitarian voting (UV) and approval voting (AV), and, therefore, “voting” and “choosing” are used interchangeably for the purposes of this paper. Also the words “alternative” and “candidate” will be used interchangeably. In Social Choice and Individual Values, Arrow (1951) clearly intends to incorporate both political and economic decision making in his analysis. Political decision making can be characterized by a social decision that applies to everyone while economic decision making can be characterized by a social decision that is comprised of individual outcomes for everyone. Arrow sets up the problem so that each individual chooser orders or ranks all alternatives and then society is required to come up with an ordering that is best according to his stated criteria. He states (Arrow, 1951: p. 11-12) “In the theory of consumer's choice each alternative would be a commodity bundle; ... in welfare economics, each alternative would be a distribution of commodities and labor requirements. … in the theory of elections, the alternatives are candidates.” In today's economy rather than alternatives being commodity bundles, they might instead be cash payments and labor requirements.

Claude Hillinger (2005: pp. 295-321) has made the case for utilitarian voting:

“There is, however, another branch of collective choice theory, namely utilitarian collective choice, that, instead of fiddling with Arrow’s axioms, challenges the very framework within which those axioms are expressed. Arrow's framework is *ordinal* in the sense that it assumes that only the information provided by individual orderings over the alternatives are relevant for the determination of a social ordering. Utilitarian collective choice assumes that individual preferences are given as *cardinal* numbers; social preference is defined as the sum of these numbers."

The difference between Hillinger's statement and the mechanism considered here is that social preference is *not* defined as the sum of cardinal numbers. There is a unique transformation done by the mechanism itself *for each voter* from their cardinal inputs to their AV style contribution to the social choice output. Hence, the system we examine is a utilitarian approval hybrid (UAV).

Lehtinen (2015: p.35) has shown that "strategic behavior increases the frequency with which the *utilitarian winner* is chosen compared to sincere behavior ". The utilitarian winner is the one that maximizes the social utility of the social choice. Therefore, the mechanism described in this paper should accomplish two things: sincere voting behavior on the part of individuals *and* increased selection of the utilitarian winner or winners compared to other voting systems. While Lehtinen abandons the Arrow and Gibbard-Sattherwaite conditions in the interests of increased social utility, strategyproofness is not violated if the mechanism, which amalgamates the individual voting/choosing information, itself applies the strategy instead of the individual choosers. It is shown elsewhere (Lawrence, 2024: pp. 9-11) that in fact the Optimal

Choice Mechanism (OCM) considered here does in fact result in the utilitarian winner(s). Therefore, it is possible to specify a maximin condition which raises the utilities of the least well off while lowering somewhat the utilities of the utilitarian winner(s).

Lehtinen (2015: p. 39) also argues that interpersonal comparisons "can be made in a methodologically acceptable way in evaluating the performance of voting rules if the same comparison is made under every voting rule." The issue of interpersonal comparisons is demonstrably moot for the implementation of the social choice mechanism considered here because an affine linear transformation of each individual's utilitarian style input does not affect their contribution to the social choice results.

The method constructed in this paper inputs information from the individual choosers in the form of preference ratings over each alternative and outputs information in the form of complete social preference rankings of the alternatives from which social ratings can be derived since the underlying individual ratings are known. From these social preference rankings, an unordered winning set, *W,* of size *m*, is constructed consisting of those alternatives with the top *m* rankings. In the case of elections the winning set would be the members of a legislative body or in the case of m = 1, a President. In the case of consumer's choice we assume that each consumer would submit a list of all available commodities with their utilities associated with each item on the list. Then the winning set would consist of a set of commodities available to all consumers. Further refinements of the theory might individualize this set for each consumer. In the case of welfare economics, rather than being a distribution of commodities and labor requirements as Arrow suggests, the more likely scenario would be a distribution of cash payments and labor requirements. The utility of the winning set for each voter/consumer, which is the summation of utilities over each alternative in the winning set divided by m, can be computed since we know from the individual inputs how each individual rated each alternative. Summing utilities over all voter/consumers and dividing by q gives the social utility of the winning set.

In order to overcome the Gibbard-Sattertwaite theorems, which maintain that every choosing system for which an individual chooser can use strategy to improve the outcome for themselves violates Arrow's conditions, we choose a social choice mechanism which itself implements the optimum strategy for each individual assuming that that strategy consists of each individual's choosing in such a way as to maximize the expected utility of their contribution to the winning set. We assume a completely random distribution of chooser preferences so that each chooser has no knowledge of the utility profiles of other choosers. The optimum strategy can be known both to the individual chooser and to the mechanism which amalgamates the choices itself. If the mechanism applies the optimum strategy to each chooser's input, then the individual chooser is disincentivized from doing so and is incentivized to submit their true utilities.

Each chooser rates each candidate by assigning to him or her a real number between zero and one. The utility profile, U, consists of this set of ratings. The mechanism described here involves the placing of an individualized threshold in the monotonically increasing and unrestricted utility profile which is submitted by each individual chooser. Each candidate above this threshold is given an approval style vote of "+1", and each candidate below threshold is given an approval style vote of "0". This strategy can be seen as the extension of the strategy when there are only two candidates which is to give the one with higher utility an approval style vote of "1" and the one with lower utility an approval style vote of "0." The threshold is placed such that the collective average utility above threshold is a maximum. When there are more than two candidates and $m \geq 1$, we vote approval style and give multiple candidates a rating of "1' and the rest a rating of "0". In approval voting (AV) the placing of the threshold is left up to the voter. In this paper we calculate a more exact rationale for placing the threshold taking into account the probability of being elected to the winning set for each candidate, and other factors such as the number of candidates and the size of the winning set in order to maximize the expected value of utility of each individual's contribution to the winning set. Heuristically, the threshold should be set higher in an individual's utility profile if the ratio of m to n is small and lower if the ratio of m to n is large. Also the utilities that each voter has for the candidates should be mitigated by the probabilities that those candidates will actually be elected. Those probabilities are set by the voters as a whole, and they must be used in the analysis. If the set of candidates above threshold has a low probability of being elected, then perhaps the threshold should be placed somewhere else.

As the threshold increases, there are less candidates above threshold, the average utility rating of the set of candidates above threshold increases, and the probability of selecting randomly any particular candidate in this set decreases. Conversely, as the threshold decreases, the number of candidates above threshold and the probability of random selection of one of them increases while the average utility rating of that set of candidates decreases. The mechanism we explore here chooses the optimum threshold, individualized for each chooser, to be just under that utility such that the expected value of their contribution to the winning set, W, is a maximum.

Formal Statement of System Parameters

We first define the following sets:

i) $V = \{v_1, v_2, \ldots v_q\}$ is a set of choosers of size q, where $v_j \in V$ denotes the j^{th} chooser.

ii) $C = \{c_1, c_2, \ldots, c_n\}$ is an ordered set of candidates of size n; candidates appear on the ballot in $c_1, c_2, \ldots, c_n$ order. $c_i \in C$ denotes the i^{th} candidate.

iii) $X = \{x_1, x_2, \ldots x_n\}$ $x_i = \{\mathbb{N}^0\}$, is a set of non-negative integers. X represents the cumulative votes for candidates as they appear on the ballot.

iv) $Y = \{y_1, y_2, \ldots y_n\}$ is the set which orders the candidates by the number of votes received by each candidate. $y_1 R y_2 R \ldots R y_n$.

v) $W = \{w_1, w_2, \ldots, w_m\}$ is a set of candidates of size $m < n$ representing the unordered winning set.

vi) $C_j = \{c_{1j}, c_{2j}, \ldots c_{nj}\}$ is the ordered set of preferences for alternatives of the j^{th} voter.

$c_{1j} R_j c_{2j} R_j c_{3j} R_j c_{4j}, \ldots, c_{n-1j} R_j c_{nj}$

vii) $B_j = \{b_{1j}, b_{2j}, \ldots, ,b_{nj}\}$ is a set of approval style votes in order of the j^{th} voter's candidate preferences. $b_{ij} = \{ \mathbb{N}^0 \mid 0, 1\}$

viii) $O_j = \{o_{1j}, o_{2j}, \ldots\}$ O_j is the set of candidates given approval style votes of "1" by voter j, called the optimal set.

ix) $U_j = \{u_{1j}, u_{2j}, \ldots u_{nj}\}$ is a set of utilities of size n, with $u_{1j} \geq u_{2j} \geq \ldots \geq u_{nj}$ and $0 \leq u_{ij} \leq 1, \forall$ i,j. U_j is the utility set of the j^{th} voter after applying an affine linear transformation to their submitted set of utilities. u_{ij} is the utility of candidate c_{ij}

x) $T_j = \{t_{1j}, t_{2j}, \ldots t_{nj}\}$ is a set of thresholds of size n such that $t_{1j} \geq t_{2j} \geq \ldots \geq t_{nj}$ and $0 \leq t_{ij} \leq 1, \forall i, j$.

xi) $U_{aj} = \{u_{a1j}, u_{a2j}, \ldots, u_{anj}\}$ is the set of utilities above threshold for each chooser. u_{aij} is defined as the sum of utilities above threshold t_{ij} for voter j, $\forall$ i,j . The sum of utilities above threshold is computed for each of the n thresholds. n_{aij} is the corresponding number of utilities above threshold $\forall i, j$. u_{aij}/n_{aij} is the average utility above threshold.

We now define following functions:

i) $\tau : C \rightarrow X$ defines an ordered pair, (c_i, x_i) such that $\tau(c_i) = x_i$,the cumulative number of votes for each candidate.

ii) $\alpha : X \rightarrow Y$ α defines an ordered pair (x_r, y_r) such that $[\, y_r R y_z$ iff $x_r \geq x_z \,]$

for $1 \leq r, z \leq n$; r, z, n *integers.*

iii) $\beta : Y \rightarrow W$ such that $\beta(y_i) = w_i$ for $1 \leq i \leq m$. The function, β, places the top m vote getters in the winning set. If y_m represents a tie with y_{m+z} for $z \geq 1$, ties are resolved randomly so that W is always of size m.

iv) $\chi_j : C \rightarrow C_j$ The function χ_j assigns to each element $c_i \in C$ an element $\chi_j(c_i) = c_{ij}$ such that $c_{1j} \mathrm{R}_j c_{2j} \ldots c_{(n-1)j} \mathrm{R}_j c_{nj}$ for $1 \leq j \leq q$ where R_j means "is preferred or indifferent to". Each voter, j, orders the set of alternatives according to their preferences.

v) $\eta_j : C_j \rightarrow U_j$ the function η_j assigns to each element $c_{ij} \in C_j$ an element $\eta_j(c_{ij}) = u_{ij}$ where u_{ij} is the utility that is assigned to candidate c_{ij} by voter j.

vi) $\delta_j : C_j \rightarrow B_j$ defines an ordered pair (c_{ij}, b_{ij}) such that $\delta_j(c_{ij}) = b_{ij}$ for $1 \leq j \leq q$ and $1 \leq i \leq n$

vii) $\gamma_j : U_j \rightarrow T_j$ defines the relationship $\gamma_j(u_{ij}) = t_{ij}$ such that $t_{ij} = u_{ij} - \varepsilon$ where $\varepsilon \ll 1$, $\forall\, i, j$

viii) $\phi_{aj} : T_j \rightarrow U_{aj}$ such that $\phi_{aj}(t_{ij}) = u_{aij}$, where $u_{aij} = \sum_{u_{ij} > t_{ij}} u_{ij} \quad \forall\, i, j$

Strategy

We focus now on one particular voter called the focal voter. While Brams and Fishburn (1983: p. 73) "presume that voters' preferences are more or less evenly distributed over the different preference orders for the ... candidates," we determine the best way for a voter with a particular preference order, in this case a utility profile, to vote. We analyze the focal voter's efficacy in changing the election results using strategy due to their choice alone. We assume that this focal voter has a utility profile, U_j, and the election is multi-candidate and multi-winner. The focal voter is interested in maximizing the utility of the winning set, W, for theirself. The strategy involves separating the candidates into two dichotomous sets by placing a threshold in the focal voter's set of utilities such that approval style votes of "+1" are cast for utilities above threshold and approval style votes of "0" are cast for utilities below that threshold.

We model the situation as a ball and urn problem consisting of n black and white balls representing the candidates. We identify the white balls with candidates above threshold and black balls with candidates below threshold. Let n_{aij} be the number of candidates above threshold and $n - n_{aij}$ be the number of candidates below threshold. We choose randomly m balls out of the urn without replacement and place them in the winning set, W. The probability, p, of k above threshold candidates being in the winning set due to chance alone is given by the hypergeometric function:

$$p = \frac{\binom{n_{aij}}{k}\binom{n - n_{aij}}{m - k}}{\binom{n}{m}}$$

We assume no prior knowledge or polling information regarding candidate probabilities although the analysis can be generalized to the case where polling information is available. Exactly which white ball (associated with a particular candidate) is picked is not known. However, the average utility of above threshold candidates, u_{aij}/n_{aij}, can be calculated.

Let u_{Wj} be a random variable which represents the average utility of above threshold candidates in the winning set due to voter j's choice alone so that $0 \leq u_{Wj} \leq 1$, $\forall j$. Then the expected value of average utility of above threshold candidates in the winning set for voter j at threshold t_{ij} is given by

$$E_{t_{ij}}(u_{w_j}) = \sum_{k=1}^{s} \left\{ \left[\frac{\binom{n_{aij}}{k}\binom{n-n_{aij}}{m-k}}{\binom{n}{m}} \right] \left[\frac{u_{aij}}{n_{aij}} \right] \right\}$$

where $s = min\{ m, n_{aij}\}$

We now perform a thought experiment in which we vary n_{aij} from 1 to n. For each value of n_{aij} we randomly withdraw balls from the urn and place them in the winning set. We do this repeatedly to determine the value of expected utility at each threshold, $t_{ij} = u_{ij} - \varepsilon$ where $\varepsilon \ll 1$. Let t_j^* be the optimal threshold which is the threshold which results in the maximization of the expected value of average utility, $Et_{ij}(u_{Wj})$, for voter j. n_j^* is the corresponding number of candidates with utilities above that threshold.

So

$$E_{t_j^*}(u_{w_j}) = \max\{E_{t_{ij}}(u_{w_j})\}$$

The set of candidates above optimal threshold is called the optimal set, O_j.

$O_j = \delta^{-1}{}_j(B_{ij})$ such that $b_{ij} = 1$. As the threshold is decreased from $t_j^{\star}$, the average utility of the optimal set for voter j decreases because there are more above threshold utilities with lower values of utility under consideration, and the probability of an above threshold candidate being in the winning set increases. As the threshold is increased from $t_j^{\star}$, the probability of an above threshold candidate being in the winning set decreases, and the average utility of the set of candidates above threshold increases. Candidates whose utilities are greater than the optimal threshold, $t_j^{\star}$, will be given the maximum AV vote of "+1", and candidates whose utilities are less than $t_j^{\star}$ will be given the minimum AV vote of "0" $\forall j$. The individual voter's strategy is to give a one vote boost to candidates above threshold, which belong to the set for which the voter has the greatest expected average utility.

If the ball and urn experiment were to be performed on each member of the electorate as a whole minus the focal voter, the total number of white balls representing AV votes for each candidate could be computed. With the addition of the focal voter's votes, there is a finite probability that one or more candidates would be elevated to the winning set resulting in a tie or ties with a candidate already in the winning set. The focal voter's AV votes could potentially determine the constitution of the winning set if a candidate's being in the winning set can be determined by single votes after all other voters have cast their ballots. If ties are resolved randomly, the focal voter could still determine the constitution of the winning set.

Counting the Votes

The vote count proceeds by the following algorithm, **σ**:

σ: for z = 1, n
    $x_z = 0$
end z (initializes X)

```
for j = 1,q
   for i = 1, n
      b_ij = 0
      if {u_ij ≥ t_j*} then
         b_ij = 1
         x_i = x_i + 1
   end i
end j
end σ
```

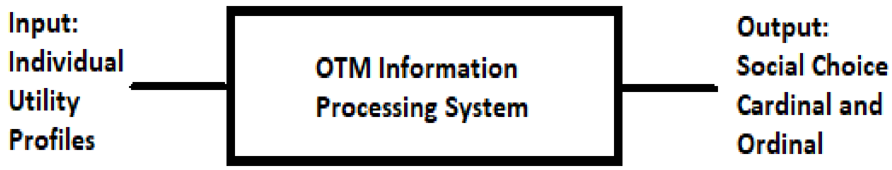
Let ${}^{A}u_j$ be the utility of the winning set, *W*, for voter/chooser *j* post-election, and ${}^{A}u$ be the social utility of the winning set for all voter/choosers - the utility of the social choice. It is also possible to compute individual and social utilities based on the voters' original utility profiles before the affine linear transformation to $0 \leq u_{ij} \leq 1$.

$$ {}^{A}u_j = \sum_{i=1}^{m} \eta_j \chi_j \tau_i^{-1} \alpha_i^{-1} \beta_i^{-1}(w_i) \qquad {}^{A}\mathbf{u} = \sum_{j=1}^{q} {}^{A}\mathbf{u}_j $$

Optimal Threshold Mechanism

The Optimal Threshold Mechanism (OTM) Information Processing System can be modeled as follows:

Figure 1



The OTM mechanism uses the above analysis to optimize each individual's choice so that they are disincentivized from choosing insincerely. It overcomes Gibbard-Satterthwaite's concerns about strategic choosing by individuals while meeting Arrow's rational and normative conditions as proven below. It even upgrades Arrow's normative conditions since more finely tuned cardinal input information is used while Arrow's analysis only involved less precise ordinal information. Moreover, the welfare or utilitarian results for each individual and for society as a whole are measurable. The key is that individuals are disincentivized from voting insincerely because the OTM system strategizes for them. The optimal strategy maximizes the expected value of utility of the winning set, *W*, for each voter/chooser based on their vote/choice alone. The assumption of utility maximizing is made by other writers (Lehtinen, 2008: pp. 688-704): "Under strategic behaviour voters are assumed to maximise expected utility ... ". The voter's input is the ordered set of candidates C_j and the associated ordered set of utilities U_j. The output is the set *Y* consisting of the ordered set of all candidates by vote totals from which is derived the winning set, *W*, which is unordered and consists of $m < n$ candidates. It is assumed that each individual voter specifies an unrestricted, utilitarian style input profile which represents their sincere utility ratings for candidates in the set, *C*.

Examples

We have computed the expected average utility vs threshold for individual utility profiles U1 and U2 (dropping the j). We have plotted $E_{tij}(u_{Wj})$ (simplifying notation to E(u) vs threshold T) for

$n = 21$, $1 \le i \le 21$, $m = 1 - 4$ as shown in Figures 2 and 3. Figure 2 represents a "smooth transition" between utilities. Figure 3 represents an "abrupt transition" between utilities.

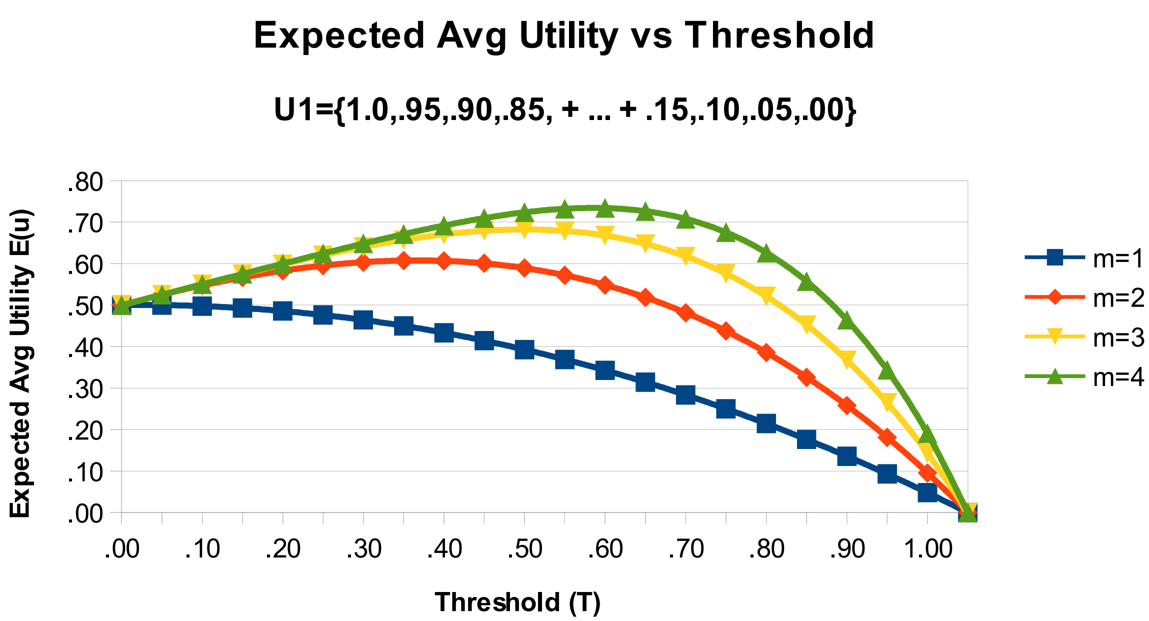


Figure 2

For m=1, E(u) max = .5000 @ T = 0.05. For m=2, E(u) max = .6075 @ T = 0.35.
For m=3, E(u) max = .6823 @ T = 0.50. For m=4, E(u) max =.7338 @ T = 0.60

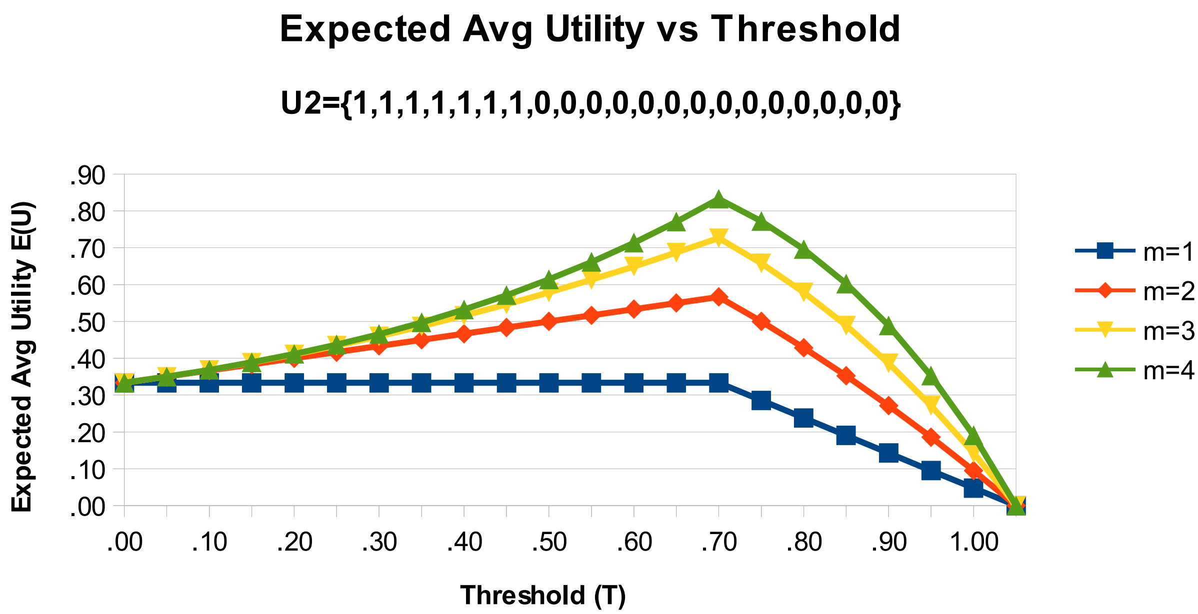


Figure 3

For m=1, E(u) max = 0.3333 @ T = 0.7. For m=2, E(u) max = 0.5667 @ T = 0.7.
For m=3, E(u) max = 0.7263 @ T = 0.7. For m=4, E(u) max = 0.8327 @ T = 0.7.

We consider t_j^* to be the greatest value of T such that *E(u)* is a maximum i.e. limsup[*E(u)*] for

0 < T < 1, as shown in Figure 3 for *m* = 1. Figure 3 shows that the optimal threshold for this utility profile is always at 0.7 regardless of the value of *m* which is intuitively plausible. As *m* increases, the expected average utility of the social choice for an individual with this profile approaches +1.

Figure 2 shows that for utility profile U1 and *m* = 1 the best strategy is to give an approval style vote of "1" to all candidates except the one whose utility is "0". That one gets an approval style vote of "0". As the size of the winning set increases, however, fewer candidates are assigned an approval style vote of "+1", and the expected average utility of the winning set for the voter with this utility profile increases.

With regard to approval voting, Smith (2005) proves the following: “Mean-based thresholding is optimal range-voting strategy in the limit of a large number of other voters, each random independent full-range.” Range voting is similar to utilitarian voting. While Smith's analysis assumes a completely randomized set of utility profiles, it does not give the optimal strategy for any particular utility profile. Lehtinen (2010: pp. 285-310) has also used expected utility maximizing voting behavior to indicate which candidates should be given an approval style vote. He agrees with Smith that an approval style vote of "+1" should be given to all candidates for whom their utility exceeds the average utility of all candidates and a "0" otherwise. Brams and Fishburn (1983: p. 90) also agree with Smith and Lehtinen: "When cardinal utilities are associated with the preferences of a voter, his utility-maximizing strategy in large electorates is to vote for all candidates whose utilities exceed his average utility over all the candidates." These writers consider only single member districts.

Based on the examples in Figues 2 and 3 we would disagree.With regard to Figure 2, the average utility for a voter with profile U1 is 0.5, but our results show a maximum expected average utility at a threshold of 0.05 for $m = 1$, and progressively higher optimal thresholds for higher values of m. For Figure 3 the optimal threshold is 0.7 for all values of *m* with maximum expected average utility increasing as *m* increases. The average utility for U2 is 7/21 = 0.33. If the threshold for "+1" approval votes with *m* = 1 were to be set to 0.33 as the above writers suggest, the expected average utility would be the same as what it is at the optimal threshold of 0.7, but more candidates with utility values of zero would be given approval votes of "1".

Proof of Theorem: The OTSC Mechanism Satisfies Arrow's Five Conditions

Arrow's five rational and normative conditions are

1) Unrestricted domain.

2) Positive Association of Individual and Social Values

3) Independence of Irrelevant Alternatives (IIA)

4) Citizens' Sovereignty

5) Non-dictatorship

Lemma 1

xR_jy iff $u_{xj} \geq u_{yj}$, by definition

xR_jy iff $b_{xj} \geq b_{yj}$, by definition

xP_jy iff $u_{xj} > u_{yj}$, by definition

xP_jy iff $b_{xj} > b_{yj}$, by definition

Lemma 2

With reference to algorithm **σ,** $b_{ij} = 1$ iff $u_{ij} \geq t_j^*$. $b_{ij} = 0$ iff $u_{ij} < t_j^*$.

$b_{xj} = 1 \wedge b_{yj} = 0$ iff $u_{xj} \geq t_j^* \wedge u_{yj} < t_j^*$

$b_{xj} = 0 \wedge b_{yj} = 0$ iff $u_{xj} \wedge u_{yj} < t_j^*$

$b_{xj} = 1 \wedge b_{yj} = 1$ iff $u_{xj} \wedge u_{yj} \geq t_j^*$

$b_{xj} = 0, b_{yj} = 1$ iff $u_{xj} < t_j^* \wedge u_{yj} \geq t_j^*$

b_{xj} = AV style vote for x in U_j

b'_{xj} = AV style vote for x in U'_j

$b_{y'j}$ = AV style vote for y' in U_j

$b'_{y'j}$ = AV style vote for y' in U'_j

Proof of Condition 1: Unrestricted Domain

By assumption any alternative, c_{ij}, can be given any utility rating, $u_{ij} \in \mathbb{R}^+ \ \forall$ i,j. Neutrality is assumed with respect to the alternatives. The OTSC mechanism, R, is neutral if it treats all the alternatives the same. R is neutral if for every permutation, ψ, of the set of alternatives, C, $R[\psi(c_1), ..., \psi(c_n)] = \psi[R(c_1, ..., c_n)]$. According to Fleurbaey and Hammond (2004: p.37) Cardinal Full Comparability (CFC) asserts that an affine linear transformation so that $0 \leq u_{ij} \leq 1$, which is the assumed input to the OTSC system, will not change the results. Any affine linear transformation of a chooser's utility profile will yield the same social choice results since the optimal threshold is a function of n_j*. Without loss of generality, the OTSC system will preprocess the input utility profile and perform the affine linear transformation.

Lemma 3

For the purposes of the proof of Condition 2, we change our notation to the notation Arrow uses. x, y, x' and y' become specific to Arrow's statement of the problem and not the same as the notation used previously in this paper.

Let $u_{1j}, u_{2j}, ... u_{nj}$ and $u'_{1j}, u'_{2j}, ... u'_{nj}$ be two sets of utility profiles corresponding to the two sets of ordering relations, $R_1,..., R_j, ..., R_n$ and $R'_1,..., R'_j, ..., R'_n$ with $u_{1j} \geq u_{2j} \geq ... \geq u_{ij} \geq ... \geq u_{nj}$ and $u'_{1j} \geq u'_{2j} \geq ... \geq u'_{ij} \geq ... \geq u'_{nj}$. In terms of the OTSC mechanism we have $c_{1j} R c_{2j} R c_{3j} R c_{4j}, ..., c_{xj} R c_{x+1j}, ..., c_{yj} R c_{y+1j}, ..., c_{n-1j} R c_{nj}$ and $c'_{1j} R' c'_{2j} R' c'_{3j} R' c'_{4j}, ..., c'_{x'j} R' c'_{x'+1j}, ..., c'_{y'j} R' c'_{y'+1j}, ..., c'_{n-1j} R' c'_{nj}$. Let c_{xj} = x and c_{yj} = y; Let $c'_{x'j}$ = x' and $c'_{y'j}$ = y'.

Proof of Condition 2: Positive Association of Social and Individual Values

Statement of Condition 2: Let $R_1,\ldots, R_j, \ldots, R_n$ and $R'_1,\ldots, R'_j, \ldots, R'_n$ be two sets of individual ordering relations, R and R' the corresponding social orderings, and P and P' the corresponding social preference relations. Suppose that for each *j* the two individual ordering relations are connected in the following ways: for x' and y' distinct from a given alternative x, $x'R'_jy'$ if and only if $x'R_jy'$; for all y', xR_jy' implies xR'_jy'; for all y', xP_jy' implies xP'_jy'. Then if xPy', xP'y'.

Proof:

By assumption, $x'R'_jy'$ if and only if $x'R_jy'$

$\therefore$ By Lemma 1, $b'_{x'j} \geq b_{y'j}$ iff $b_{x'j} \geq b_{y'j}$

By Lemma 2,

$b'_{x'j} = 1 \wedge b'_{y'j} = 0$ iff $b_{x'j} = 1 \wedge b_{y'j} = 0$

$b'_{x'j} = 1 \wedge b'_{y'j} = 1$ iff $b_{x'j} = 1 \wedge b_{y'j} = 1$

$b'_{x'j} = 0 \wedge b'_{y'j} = 0$ iff $b_{x'j} = 0 \wedge b_{y'j} = 0$

$\therefore \neg(b'_{y'j} = 1 \wedge b'_{x'j} = 0)$ iff $\neg(b_{y'j} = 1 \wedge b_{x'j} = 0)$

$$\therefore \sum_j b_{x'j} = \sum_j b'_{x'j} \;\forall\, x', j$$

$$\therefore \sum_j b_{y'j} = \sum_j b'_{y'j} \;\forall\, y', j$$

$$\textbf{xP'y iff } \sum_j b_{xj} > \sum_j b_{y'j} \;\forall\, y', j$$

$$\textbf{xP'y' iff } \sum_j b'_{xj} > \sum_j b'_{y'j} \;\forall\, y', j$$

By assumption, If $xP_j\,y'$, then xP'_jy' $\forall$ y',j

$$\textbf{If } b_{xj} > b_{y'j} \textbf{ then } b'_{xj} > b'_{y'j}$$

$$\textbf{If } \left(\sum_j b_{xj} > \sum_j b_{y'j}\right) \textbf{then } \left(\sum_j b'_{xj} > \sum_j b'_{y'j}\right)$$

$\therefore$ If xPy' then xP'y' Q.E.D.

Discussion:

Condition (2) is satisfied because raising some alternative's utility, u_{ij}, in an individual's utilitarian style input from just under to just above optimal threshold will result in that alternative's receiving one more approval style choice, b_{ij}, in the final summation, *X*. This would raise the social choice result by one for that alternative potentially putting that alternative in the winning set and/or changing the ordering in the set, *Y*. Similarly, lowering a candidate's rating in some individual's utility scale might eliminate that alternative from the winning set or change the ordering of the set, *Y*.

Proof of Condition 3: Independence of Irrelevant Alternatives (IIA)

We state Arrow's Condition 3 as follows:

Let $R_1,..., R_j ,..., R_n$ and $R'_1,..., R'_j ,..., R'_n$ be two sets of individual orderings and let C(S) and C'(S) be the corresponding social choice functions. If, for all individuals j and all x and y in a given environment S, xR_jy if and only if xR'_jy, then C(S) and C'(S) are the same (independence or irrelevant alternatives).

Proof:

Let S = {x,y}

To prove: C(S) = C'(S) = x $\forall$ x,y

By assumption, xR_jy iff xR'_jy $\forall$ j

By Lemmas 1 and 2, $\lceil(b_{xj} = 0 \wedge b_{yj} =1)$ iff $\lceil(b'_{xj} = 0 \wedge b'_{yj} =1)$ $\forall$ x,y,j

$\therefore \lceil yPx$ iff $\lceil yP'x \; \forall \; x,y$

$\therefore \; C(S) = C'(S) = x$

Q.E.D.

Discussion:

Utilitarian style sincere ratings for each candidate are assumed to be independent of each other regardless of the composition of the alternative set. (Hillinger, 2004: p. 3), "A cardinal number assigned to an object indicates its place on a scale that is independent of other objects." So if an individual rates a candidate at a particular rating on their utility scale, and then another candidate enters or leaves the race, it is assumed that the first candidate will still be rated the same. A candidate's dropping out or entering the race is assumed not to change an individual's sincere ratings for the other candidates.

Now consider the case in which, after the election occurs, a candidate dies or drops out.
Arrow (1951: p. 26) states : “Suppose that an election is held, with a certain number of candidates in the field, each individual filing his list of preferences, and then one of the candidates dies. Surely the social choice should be made by taking each of the individual's preference lists, blotting out completely the dead candidate's name, and considering only the orderings of the remaining candidates in going through the procedure of determining a winner.” Arrow implies that the voting has already occurred, but the final determination of the winner(s) has not been made. If this were the case, the OTM system would blot out the dead candidate's rating from all of the individual rating scales, recompute all the individual thresholds, and recompute the ordered outcome, *Y*, and the winning set, *W*. Therefore, the dead candidate is not irrelevant, just not included in the final computation.

Now consider the case in which a new candidate enters the race after the balloting has occurred but before the election results have been published. The added utility rating for that candidate would be uploaded to the OTM system by each individual chooser after the utilities for the other candidates had presumably already been submitted, and the results had already been computed. The OTM system would then recompute the individual thresholds including the added candidate's utility ratings and the final social choice results would then be recomputed. The individual choosers would not have an incentive to rate the added candidate insincerely knowing that the OTM system would give them the strategically best outcome based on the complete list of submitted utilities. Therefore, candidate add-ons would not incentivize any individual chooser to choose insincerely. Furthermore, compliance with IIA is satisfied for add-ons since ratings for two candidates at a time could be uploaded for each individual chooser with thresholds recomputed at each step or as a final step thus demonstrating that the social choice can be arrived at by pairwise comparisons which Arrow's IIA demands.

Condition 4: The Social Welfare Function Is Not imposed.

The ouput of the OTM system is solely a function of the unrestricted inputs by assumption. There are no alternatives x and y such that xRy regardless of voter inputs. The OTM system is neutral and anonymous. It treats all citizens and alternatives the same. All permutations of V, the set of voters, and C, the set of candidates, are allowed. Permutations of voters or candidates do not change the results. The OTM mechanism, R, is neutral if it treats all the alternatives the same. R is neutral if for every permutation, ψ, of the set of voters, V, $R[\psi(v_1), ..., \psi(v_n)] = \psi[R(v_1, ..., v_n)]$.

Condition 5: The Social Welfare Function Is Not To Be Dictorial

For the OTM system $C_j = \{c_{1j}, c_{2j}, ... c_{nj}\}$. All permutations of c_{ij} are allowed, ∀ i,j. Condition (5) is satisfied since the winning set is based only on individual inputs which are all treated equally. There is no voter/consumer *j* such that xRy iff xR_jy. Therefore, the OTM mechanism satisfies all five of arrow's rational and normative conditions. Q.E.D.

Optimal Threshold Social Choice is Strategyproof

Since the data is processed in an optimal manner for each individual chooser by the system itself, giving each chooser the optimal strategy, the choosers have no incentive to misrepresent their preferences or to choose insincerely. They would either choose sincerely or the OTM system might process their input in such a way as to give them a suboptimal result. There is no advantage for individuals to misrepresent their preference ratings. The choosers are disincentivized from choosing insincerely. The strategy has been placed in the processing of the choices rather than in each individual chooser's hands.

The optimum strategy for each individual is to vote in such a way as to maximize their expected utility, ${}^{A}u_j$, for the winning set. This is done by the OTM system itself by setting an optimal threshold in each individual's utility style input so that each candidate above threshold receives the maximum "vote" and every candidate below threshold receives the minimum "vote". This maximizes the expected value of utility of the social choice for each individual based on that individual's choice alone. This effectively turns the utilitarian style inputs into approval style outputs, but the connection with the underlying utilitarian basis of the system is maintained since the original utilities are known to the OTM mechanism and can be used to compute the utility of the social choice for each individual and for society as a whole.

The Issue of Interpersonal Comparisons is Moot

Arrow (1951: p. 10) dwells on the fact that individual utility scales are not compatible. He compares them with the measurement of temperature which is based on arbitrary units and the arbitrary terminal points of freezing and boiling for the Celsius scale and completely different end points for the Fahrenheit scale.

> "Even if, for some reason, we should admit the measurability of utility for an individual, there still remains the question of aggregating the individual utilities. At best, it is contended that, for an individual, their utility function is uniquely determined up to a linear transformation; we must still choose one out of the infinite family of indicators to represent the individual, and the values of the aggregate (say a sum) are dependent on how the choice is made for each individual. In general, there seems to be no method intrinsic to utility measurement which will make the choice compatible."

Arrow's analysis is based on how an outside observer would select an indicator to represent each individual. Our analysis is based on how each individual, theirself, would choose their own indicator. Therefore, our analysis is democratic rather than paternalistic. As Arrow suggests, we take into account that each individual has a unique utility function. There is no need to (Arrow: p. 12) "choose one out of the infinite family of indicators to represent the individual." Each individual gets to choose their own indicator. Let's say that, in general, utility can be measured using the set of non-negative points u_{ij} on the real line, $R_{\geq 0}$. It's up to the individual chooser where to place the points, including the end points, corresponding to the utilities of each alternative in the alternative set consisting of n alternatives, $C = \{c_1, c_2, ..., c_n\}$. Let's call the end points of some individual's utility scale u_{max} and u_{min}. This will define the scale. There needs not be an actual utility assigned to either of these end points. Since the OTM system optimizes the utility of the social choice for each individual, there would be an optimal threshold above which all utilities are changed to the maximum value and below which all utilities are converted to the minimum value.

For the OTM system in particular, the results will be the same no matter which utility scale each individual chooses since the optimal threshold is a function of n^*_j. Any affine linear transformation of a chooser's utility scale will yield the same results since n^*_j will be the same before and after the transformation due to collinearity (Wolfram MathWorld). Let an individual express their utilities on a scale of their choice on the real line. For the sake of the analysis, the OTM system preprocesses and converts each individual's input utility scale to one with end points "0" and "1" by means of an affine linear transformation. $f(u) = au + b$ with a,b integers. Let $f(u_{max}) = +1 = au_{max} + b$ and $f(u_{min}) = -1 = au_{min} + b$. It follows that $a = 2/(u_{max} - u_{min})$ and $b = -(u_{max} + u_{min})/(u_{max} - u_{min})$.

Sen (1970a) has shown that the exclusion of interpersonal comparisons of utilities can be formulated in a more subtle way without altering the validity of Arrow's theorem, by requiring instead that social preferences be invariant only to affine rescaling of utilities which he stated as Cardinal Non-Comparability (CNC): for all $U_j, U'_j \in \mathbb{R}$, one has $f(U_j) = f(U'_j)$ whenever for all $j \in V$, there are real constants κ_j and ν_j, with each $\nu_j > 0$, such that $U'_j \equiv \kappa_j + \nu_j U_j$. The social choice mechanism detailed in this paper is invariant to affine rescaling of utilities since the optimal threshold is a function of n^*_j $\forall$j.

There is a transformation from cardinal information to ordinal information since each utility profile, $\mathbf{u} = (u_1, u_2, ..., u_n)$ is converted to a vector composed of integers, $\mathbf{v} = (v_1, v_2, ..., v_n) \in \{0,1\}$. So even though the individual utilities are cardinal noncomparable, the transformed utilities are ordinal and comparable by assumption. What's more, the individual utilities and social utility of the final results are computable since the original individual utilities are known to the system. A maximin or leximin transformation of the results is also possible making cardinal full comparability unnecessary. Since we prove later that the OTM mechanism results in the social choice which maximizes social utility, (the utilitarian winner), implementing a maximin or leximin condition diminishes the utility of the social choice in order to insure that each participant has at least a minimum utility at the outcome. This compensates for the fact that as Arrow writes "The viewpoint will be taken here that interpersonal comparison of utilities has no meaning and, in fact, there is no meaning relevant to wefare comparisons in the measurability of individual utility."

Consequently, Arrow's statement that “the values of the aggregate are dependent on how the choice is made for each individual” is not true. The choice is not *made* for each individual; each individual makes their *own* choice. However, since any scale chosen by each individual will yield the same results, without loss of generality, we can standardize the choosing process by transforming individual scales to the real line between "0" and "+1", preprocessing the data before input to the OTM system.

Amartya Sen (2002: p. 71) stated “... economists came to be persuaded by arguments presented by Lionel Robbins and others (deeply influenced by "logical positivist" philosophy) that interpersonal comparisons of utility had no scientific basis. 'Every mind is inscrutable to every other mind and no common denominator of feelings is possible.' Thus, the epistemic foundations of utilitarian welfare economics were seen as incurably defective." The OTM system demonstrates that there is a sound epistemic basis for a utility based social choice mechanism. Therefore, it is in fact logical positivist *because* it has a sound scientific basis. Showing that Arrow's and Gibbard-Satterthwaite's impossibility results are invalid for just one mechanism such as OTM proves that social choice is not impossible potentially for other mechanisms as well.

"The difficult we do right now, the impossible will take a little while" (from "Crazy He Calls Me" by Carl Sigman and Bob Russell.)

Preference Rankings Can Be Converted to Ratings and Vice Versa

Arrow's assumption of input preference orderings or rankings for each individual is a tacit assumption of equal utility scales for each individual equivalent to the “one man, one vote” principle. With the assumption that individual orderings represent equally spaced utilities, we can convert orderings or rankings to ratings. This may or may not be a very accurate representation of the underlying utilities, but it's the best information available if only individual orderings are known. These ratings can then be used as inputs to the OTM mechanism.

The available information for rankings is of the form aRbRcRd... . For the system considered here and without loss of generality, any scale with any end points can be used for this conversion procedure as long as the preference ratings are equally spaced. For instance, we can choose the real line between "0" and "+1". We let the top ranked candidate be placed at "+1" and the lowest ranked candidate be placed at "0". The other candidates then would be equally spaced on the scale. The OTM information processing system will then output approval style positive choices for those candidates represented by utilities above the optimal threshold and zero choices for those candidates represented by utilities below the optimal threshold for each individual. As we have shown, any affine linear transformation of an individual's utility scale will not change the results of the OTM mechanism. The outputs are in the form of integers and represent the votes or choices for each alternative or candidate. Thus individual inputs can be in the form of rankings if utility information is not available. Therefore, the OTM inputs and outputs can both be represented as rankings (orderings) and/or ratings (utilities).

Conclusions

We have proved that the OTM mechanism satisfies Arrow's five rational and normative conditions. It has been shown that social choice is possible thus replacing both Arrow's and Gibbard-Satterthwaite's impossibility theorems which are devoid of the inherently probabilistic nature of voting/choosing methods. Their results apply to certain deterministic mathematical structures and were not extended to the more realistic probabilistic case considered here. We have developed a completely new concept, the Optimal Threshold Mechanism (OTM), which accepts Arrow's and Gibbard-Satterthwaite's conditions and yet produces actual possible results. Furthermore, since we deal with utilitarian rather than preference ordering information, the results manifest an upgraded and more robust version of Arrow's normative conditions. Utilitarian satisfaction is also measurable both at the individual and social levels after the choosing process occurs. The OTM system accepts individual utilitarian style preference ratings as inputs and outputs approval style social choice preference rankings. It processes the inputs in such a way as to maximize the expected utility of the social choice for each individual chooser based on their choices alone. This is done by setting an optimal threshold in the input utilitarian data of each individual chooser and outputting "+1" approval style choices for those candidates above threshold and "0" approval style choices for those candidates below threshold. Thus the input data is converted into approval style outputs which are then summed over all choosers. This produces social choice rankings for all of the alternatives. The optimal threshold resolves the issue in approval voting of how to accurately divide the candidates into two groups. Since the OTM system converts utilitarian style inputs to approval style outputs, OTM is a utilitarian approval hybrid (UAV) system. Although we assume no knowledge of polling statistics, the OTM system is generalizable to the case in which polling information is known.

The issue of interpersonal comparisons is moot because any affine linear transformation of an individual's utility scale will produce the same results when processed by the OTM system. If inputs are specified as preference rankings rather than ratings, the rankings can be converted to utility style ratings which can then be processed by the OTM system. The outputs which are in the form of social rankings can also be converted back to ratings because the underlying utility information for each individual chooser is known. The utility of the social choice can be computed for each individual and for society as a whole.

Finally, the OTM system will produce the utilitarian winner(s), that is the winner(s) that maximize social utility. It has been shown by other writers (Lehtinen,2015: p.35) that "strategic behavior increases the frequency with which the *utilitarian winner* is chosen compared to sincere behavior ". In the OTM system considered here, the strategy is implemented by the mechanism itself so that individual choosers have no incentive other than to input honest and sincere choices.

Arrow's main conclusion has been known since 1785 from the work of the Marquis de Condorcet, but Arrow attempted to elaborate and recast the paradox of voting as a proof that any kind of rational system which purports to determine the public good instead leads to a dictatorship which accorded nicely with Cold War philosophy directed at the Soviet Union. Alex Abella (2008: p. 49) wrote:

> “To combat the communist credo, postwar American intellectuals sought a version of history that eliminated once and for all the Marxist dogma: 'From each according to his ability, to each according to his needs.' The new doctrine would substitute the oppressive, omniscient Marxist state with a system that championed the right of individuals to make their own choices and their own mistakes. That doctrine, elaborated at RAND in 1950, was called rational choice; its main proponent, a twenty-nine-year-old economist named Kenneth Arrow."

The American and French revolutions of 1776 and 1789 respectively, although originally expressing their zeal for government by the people, ended up enshrining power in representative government precisely because the writers of their Constitutions did not trust the people. One of the most important theoreticians of the French revolution, the Abbe Sieyes, wrote (Harries-Jones, 2016: p. 78), “In a country that is not a democracy – and France cannot be one – the people, I repeat, can speak or act only through its representatives.” David Van Reybrouck writes (2016: pp. 89-91), “The French

Revolution, like the American, did not dislodge the aristocracy to replace it with a democracy but rather dislodged a hereditary aristocracy to replace it with an elected aristocracy, *'une aristocratie elective'*, to use Rousseau's term.” The impossibility theorems of Arrow and Gibbard-Satterthwaite seem to have driven this point home since they claim that economic democracy and political direct democracy are impossible leaving only capitalist economics and representative democracy with a sound epistemic basis. In the American system of democracy in particular, gerrymandering has insured that its representatives will indeed constitute *'une aristocratie elective'.* The work presented here proves that direct political and economic democracy do in fact have a sound scientific basis and that rational and normative social choice is indeed possible.

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