RATIONAL SOCIAL CHOICE IS POSSIBLE: A REFUTATION OF ARROW'S IMPOSSIBILITY THEOREM

PART 1

In 1951 Kenneth J. Arrow published "Social Choice and Individual Values"[1] in which he explored the question of whether or not individual preferences could be aggregated in some rational way in order to form a social choice. He postulated five rational and ethical criteria that such a social decision function should meet, and then proceeded to prove that no such social decision function existed. This theorem is known as Arrow's Impossibility Theorem, and an impressive literature concerning itself with what has come to be known as social choice theory has developed in the last nearly forty years. A sub-field of welfare economics has thereby been created.

Some of the literature has been concerned with finding a way around Arrow's basic result that no rational social choice is possible by relaxing one or more of his criteria.[2,3,4,5] Arrow's theorem has important political, economic and social implications since, if indeed no rational way to aggregate individual preferences is possible and Pareto optimality is the best we can do, then a populist democracy which closely reflects the will of the people becomes impossible and free market capitalism acquires a theoretically endorsed superiority over any kind of populist socialistic or democratic economy. This realization has produced pessimism and even nihilism among proponents of welfare economics.[6] However, advocates of democratic voting systems should be equally concerned as Arrow's result tarnishes the validity of democratic elections as well.[7,8]

INDIVIDUAL PREFERENCES

Let us assume we have a society composed of T individuals numbered 1 through T. For identification purposes, we can refer to the j^{th} individual. We have a set of options, S,

composed of N options, A, B, C,...etc. We presume that each individual has a set of preferences over the options and that there is a social decision function, C(S), which orders the option set according to social preference. So far we are in agreement with Arrow. But here is where we part ways: the nature of the set of preferences. Arrow assumes that they are strictly ordinal i.e. for each individual

$$A R_j B$$

means A is preferred or indifferent to B for the j^{th} individual. Therefore, for each individual we have an ordered set in which each alternative is either preferred or indifferent to every other alternative.

COMPARISON OF ORDINAL AND CARDINAL INFORMATION

We take the point of view that a voter or a consumer is capable of expressing more than just ordinal information. Since a person is capable of specifying not only the fact that he or she prefers A to B to C but also information such as "I strongly prefer A to B but am relatively indifferent between B and C although I slightly prefer B to C," it is clear that in principle each citizen is capable of expressing a fuller set of preferences than is represented by ordinal information alone. This fuller set of preferences includes preference intensity information. For the purposes of this paper we define the total information as the sum of cardinal and ordinal information where ordinal information represents rank ordering and cardinal information represents preference intensity. Thus we maintain that more relevant information can be obtained through "expressed preferences" than through "revealed preferences."[9] We wish to precisely compare and give a measure to these two types of information.

C. E. Shannon in his paper "A Mathematical Theory of Communication"[10] laid the groundwork for the essentials of information theory. In particular he defined a measure of information as

$$\log_2 M$$

where M = the number of possible states. The units of information are called bits for binary digits.

Let us consider the preference information of a single voter and an option set containing two alternatives, A and B. Also let us assume for the present discussion that there are no indifferences i.e. A or B is strictly preferred to the other. Clearly the ordinal information involved is 1 bit since there are two possible states, A P_j B or B P_j A.

When we consider cardinal information, we must ask the question: How much is A or B preferred to the other? In general, if we assume the existence of two poles - the ideal best and ideal worst options - then A and B will be somewhere between the two poles as in Figure 1.

FIGURE 1 Here

If we are only concerned with preference intensity information, then we are only interested in the distance between A and B and not their absolute placement with respect to the poles. Therefore, without loss of generality, we can put the least preferred alternative at one pole and let the most preferred alternative assume values between the two poles as in Figure 2.

FIGURE 2 Here

The number of possible states between the two poles will determine the information available concerning A and B. Let us assume that A Pj B and further that there are M possible states that A could be in as in Figure 3 with $\log_2 M$ being an integer. The total information regarding A and B is

$$\log_2 2M = 1 + \log_2 M.$$

The ordinal information is 1 bit and the cardinal information is $\log_2 M$ bits.

Now let us consider the case of three options: A, B and C. There are 3! or 6 possible orderings. Therefore,

$$\text{ordinal information} = \log_2 3!$$

Let us assume A P_j B P_j C as in Figure 4.

Let us assume M-2 possible states for B. Then the total information is

$$\log_2 (M-2)3!$$

The cardinal information is $\log_2 (M-2)$. The ratio of ordinal information to cardinal information is

$$\log_2 3!/\log_2 (M-2)$$

If M = 8, then half the total information will be ordinal and half will be cardinal.

Now let us assume that there are N alternatives and M different states with no indifferences. The total information is then

$$\log_2 (M!/(M-N)!$$

when the end states are assumed to be the ideal best and ideal worst states (Case 1) and

$$\log_2 [N(N-1)(M-2)!/(M-N)!]$$

when the end states are assumed to correspond to one of the actual alternatives (Case 2).

Ordinal information = $\log_2 N!$ and cardinal information = $\log_2 M!/N!(M-N)!$, (Case 1) and $\log_2 N(N-1)(M-2)!/N!(M-N)!$, (Case 2), $M \geq N$.

Note that when M =N, the total information equals the ordinal information which is to be expected and the cardinal information equals 0. Also when M » N, most of the information is cardinal information.

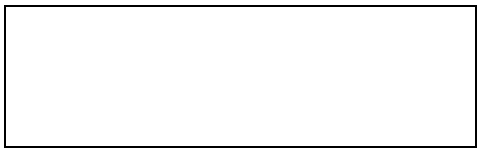
When indifferences are allowed the total information in Case 1 is

$$\log_2 M^N$$

and in Case 2,

$$N(N-1)(M-2)^{(N-2)}.$$

The ordinal information is equal to the logarithm of the number of ways, N_0, that N alternatives can be ordered with indifferences which is equal to the number of ways N alternatives can be put into M slots (M < N) such that there are no empty slots.[11]



The ordinal information, $\log_2 N_0$, satisfies the following inequality:

$$\log_2 N! < \log_2 N_0 < N\log_2 N$$

The left hand member of the inequality is the information contained in ordinal preference information withouit indifferences, and the right hand member is the information involved in placing N balls in N slots including the cases where there are empty slots which need to be eliminated. Using Stirling's approximation,

$$\log_2 N! > N\log_2 (N/e)$$

$$N\log_2 (N/e) < \log_2 N_0 < N\log_2 N$$

Let $\log_2 N_c$ = the cardinal information. Then $\log_2 N_c = N\log_2 M - \log_2 N_0$

$$N\log_2 M - N\log_2 N < \log_2 N_c < N\log_2 M - N \log_2 (N/e)$$

$$N(\log_2 M/N) < \log_2 N_c < N[\log_2 (Me)/N]$$

$$N(\log_2 M/N) < \log_2 N_c < N(\log_2 M/N) + 2N$$

From this we can see that when $M \sim N^2$, the ordinal and cardinal information are approximately equal, when $M \ll N^2$, most of the information is ordinal and when $M \gg N^2$, most of the information is cardinal. For M and N given, there is more ordinal and total information in the case where indifferences are allowed than in the case where only strict preferences are allowed.

It is worthwhile to examine Arrow's reasoning which led him to reject non-ordinal or cardinal preference information. He says: "The viewpoint will be taken here that interpersonal comparison of utilities has no meaning and, in fact, that there is no meaning relevant to welfare comparisons in the measurability of individual utility."[12] And yet it is a commonly accepted principle in democratic voting systems that the input of each individual shall have the same *weight* regardless of the needs, legitimate or otherwise, of some voters for greater

consideration. In other words irregardless of what each voter's actual utility is, in order to meet the requirement of equal inputs from all, he must standardize his preference data input so that it carries the same weight as every other voter's. The requirement of equal inputs means that individual preference data or presumed utilities, if you will, must be made to be interpersonally comparable, in the sense of overall weighting, for the purposes of democratic voting. This is done by standardizing the form of the voting data. Even in ordinalist voting schemes involving pairwise comparisons, there are interpersonal comparisons of utility when the number of voters who prefer A to B is compared to the number of voters who prefer B to A. Each voter's preference is given equal weighting and then added to the preferences of others. Thus interpersonal comparisons have taken place.

Quoting Arrow again: "Even if, for some reason, we should admit the measurability of utility for an individual, there still remains the question of aggregating the individual utilities. At best, it is contended that, for an individual, his utility function is uniquely determined up to a linear transformation; we must still choose one out of the infinite family of indicators to represent the individual, and the values of the aggregate (say a sum) are dependent on how the choice is made for each individual."[13] Let us admit that an individual's utility function (one out of the infinite family) may have some meaning to him. Then a linear transformation is applied to the utility function of each individual so that the bounds of each individual's resulting utility function are the same e.g. 0 and 1. The resultant set of utility functions or preference data contains components all of which have equal weight and, therefore, are suitable as inputs to a democratic voting system. There is no need to compare "marginal utilities at different levels of well-being" nor to set up a scale similar to a temperature scale which divides the utilities into units. Arrow continues: "It requires a definite value judgment not derivable from individual sensations to make the utilities of different individuals dimensionally compatible and a still further value judgment to aggregate them according to any particular mathematical formula."[14] He's right. There <u>are</u> two value judgments involved. The first is the equal weighting of inputs; the second is inherent in the aggregation rule e.g. maximization of social utility or some other

social decision function. Historically, when an election is decided by majority rule, some form of the maximization of social utility decision rule is used. Arrow is using value judgments to set up his five criteria. What is wrong then with using a value judgment in selecting the aggregation mechanism?

We conclude, at this point, that Arrow's arguments for confining himself to ordinal preference data are not convincing and shall proceed. In general an individual can prefer an alternative very strongly or very weakly compared to another alternative, and this intensity of preference is significant information. For example, let us assume there are two candidates A and B and 55% of the voters prefer A to B but only slightly. 45% of the voters prefer B to A very passionately. By majority rule A would win, but clearly B should win if the criterion of maximizing voter satisfaction is seriously considered. This example not only brings out the importance of preference intensity information but also casts doubt on majority rule.[15] Similar examples could be cited for three way races in which a minority candidate that was intensely disliked by the majority nevertheless won with a slight plurality by splitting the majority vote between the other two candidates as in the case of the election of Evan Mecham, the recently impeached governor of Arizona.

The key to including preference intensity information in the individual's preference ratings is to visualize the information geometrically. Each individual places his choices along a line with the distance between choices corresponding to preference intensity. The total length of the line corresponds to the intensity felt between the best and worst alternatives present in the option set. At this point there is no need to apply a numerical measure to any of the lengths involved and hence no need to consider a scale. The analysis is strictly geometric. For instance, let us consider two different option sets both of which are sets of individuals. Set 1 contains US Presidents. Set 2 contains set 1 plus Jesus and Hitler. We can imagine that most individuals would put a lot more "distance" between Jesus and Hitler than between their best-liked and least-liked US President.

Thus we have a visual geometric depiction of preference rating or presumed utility which is a more accurate description of individual preferences than is just ordinal information. This individual specification of utilities is completely general and has nothing to do with interpersonal comparisons. From this more complete informational base ordinal information is simply derived.

DEMOCRACY AND THE NORMALIZED GEOMETRIC VOTE

We will only be considering democratic social decisions i.e. each individual's preferences will be given as much weight and only as much weight in determining the outcome as any other's. This means that each individual's preference rating or utility function must be normalized in the same way that, in an election involving two candidates, each voter is allowed one and only one vote no matter how strongly he may feel about the candidates in question i.e. no matter what his actual, perceived utility function is. We can take each individual's geometric or "line" information and shrink or expand it until each line is the same length. Then each vote will have the same power or weight. For each individual each option represents a line length measured from his least-liked option. If we let the decision rule be: for each option, link the line lengths for that option from each individual voter; then each option will end up with an overall or social line length. The one with the longest social line length wins according to the maximization of social utility or Benthamite social decision function. We have not had to assign numbers or "utilities" to any of the data points. The specification of preference or utility data as well as the aggregation and determination of a winner was all done geometrically. Therefore, we have not had to attach meaning to units of utility and, consequently, Arrow's argument that we would have to have a measurable unit of utility that would be the same for all people in order to have interpersonal comparisons of utility is invalid.

Without loss of generality, we may take the one dimensional geometric information representing each individual's preference rating or utility function and place it on the real line with the left hand boundary point at 0. Applying a linear transformation if need be, we place the

right hand boundary point at 1. We now have a one to one mapping between each point on the line and a real number. We can let the real number corresponding to the placement of each option on the line be a measure of the individual's preference rating or presumed utility for that option. Then we can aggregate using the Benthamite rule of summation of utilities or the Rawlsian maximin rule or some other.

INDEPENDENCE OF IRRELEVANT ALTERNATIVES

In general if we think of the problem completely geometrically, each individual places his options along a one dimensional line that may be of any length. His most favored option will be at one end of the line and his least favored at the other. The length of the line represents his preference intensity between these two options. In the voting process the line lengths for all voters must be standardized or normalized either by the voters themselves or by the social decision function. Then the line lengths assigned by each voter to a particular option are joined together. This is done for each option, and the option with the longest line length wins.

In this process what happens if an alternative is added to or deleted from the option set? How does this affect the values assigned to the other alternatives and the ultimate outcome? Let us say an alternative is added to the option set. There are two cases here. In the first case the alternative added falls somewhere in between most favored and least favored options. It is clear that the values (line lengths) for the other options are unaffected since all values are determined with respect to the two poles. In the second case the alternative added replaces one of the poles as the most or least favored alternative for some individuals. Therefore, the individual's total line length increases. When the preference data is normalized, the line lengths associated with each option necessarily must change to accomodate the fact that one of the poles has been replaced by a new alternative. Similarly, if an option is deleted, for those individuals for whom this option represented a pole, the total line length associated with their preference or utility data would be shortened, and, after normalization, the values for the other options would be modified. These modifications in data for certain individuals could change the overall results of the social decision process. However, this comes about due to our insistence on equal input weightings from all voters. This principle takes precedence over whether or not the results of the process change when the option set changes. In other words an alternative which is added to or deleted from the option set which represents a pole for at least some of the voters is not an irrelevant alternative for those voters, but an alternative added to or deleted

from the option set which does not represent a pole for some voters is an irrelevant alternative for those voters. Since individual decisions will change depending on the option set, the social decision also may change in certain cases and so needs to be recomputed for each new option environment.

Alternatively, we can assign two arbitrary real numbers to two of the alternatives. Without loss of generality, let the numbers 0 and 1 be assigned by each voter to the worst and best alternatives in the option set respectively. Arrow seems to show that this will lead to unsatisfactory results. "Assume that for each individual there is always one alternative which is preferred or indifferent to all other conceivable alternatives and one to which all other alternatives are preferred or indifferent. Then, for each individual, the utility indicator can be defined uniquely among the previously defined class, which is unique up to a linear transformation, by assigning the utility 1 to the best conceivable alternative and 0 to the worst conceivable alternative. This assignment of values is designed to make individual utilities interpersonally comparable."[16]

What Arrow is assuming here is that the ideal best and ideal worst alternatives are contained in the actual option set, but then he goes on to cite an example in which this is not true and uses the example to argue that there is arbitrariness involved in the assignment of utilities. He continues: "It is not hard to see that the suggested assignment of utilities is extremely unsatisfactory. Suppose that there are altogether three alternatives and three individuals. Let two of the individuals have the utility 1 for alternative x, .9 for y, and 0 for z; and let the third individual have the utility 1 for y, .5 for x and 0 for z. According to the above criterion, y is preferred to x. Clearly, z is a very undesirable alternative since each individual regards it as worst. If z were blotted out of existence, it should not make any difference to the final outcome; yet, under the proposed rule for assigning utilities to alternatives, doing so would cause the first two individuals to have utility 1 for x and 0 for y, while the third individual has utility 0 for x and 1 for y, so that the ordering by sum of utilities would cause x to be preferred to y."[17]

Arrow's problem here is that he doesn't follow the assignment rule he, himself, suggests which is to give utility 1 to the best conceivable alternative and 0 to the worst conceivable alternative. If 1 is assigned to the best conceivable alternative and 0 to the worst conceivable

alternative, then all comparisons of actual alternatives are with these two poles which remain fixed regardless of the size of the option set and hence all ratings are non-arbitrary. Clearly, if we assume the worst ideal, z, is in the set even though it may not be a viable alternative, then the ratings for x and y do not change and the ratings lose their arbitrariness. Therefore, if each individual assigns the values 0 and 1 to his worst and best conceivable ideals and adds them to the set for the purpose of determining the ratings of the other options, even though these ideals may not be actual options in the sense that they can be chosen, the ratings assigned will not be arbitrary regardless of whether or not certain alternatives are added to or deleted from the set. In this case any alternative added to or deleted from the option set will be irrelevant in the sense that none of the individual values for other alternatives will change and, consequently, the overall social results will not change either.

It is important that the voters choose their ideal best and worst alternatives carefully. A problem might arise if each individual chose their best and worst ideals according to different criteria. For example, one voter's ideals might be so far removed from the actual candidates as to cluster all preference ratings for the actual candidates close together as in Figure 5.

FIGURE 5 Here

Another voter might identify his ideals more with the best and worst actual candidates so that his preference rating might look like this:

FIGURE 6 Here

Actual candidates would be placed at 0 and 1. The problem here is that Figure 6's voter's preferences would carry more weight than Figure 5's and hence Figure 5's voter's input would tend to be discounted. This can be remedied by giving the voters instructions as to how to select their ideals. If each voter selects his ideals according to the same criteria, then there is no problem. If we are selecting a candidate for a representative, legislative body, our best ideal should be one who would vote exactly as we would always; the worst ideal would vote as we would never. For each actual candidate, we would have to judge on what percentage of the issues he would vote as we would. Then the preference ratings would all have equal voting power and be non-arbitrary as long as voters voted sincerely, that is, they did not misrepresent themselves. Practically speaking, we could look at, say, a number of issues and examine a candidate's voting record on each. We know how we would have voted on each of the issues. If the candidate voted as we would have on 60% of the issues, then we would assign him a utility of .6. Spatial models to determine our "distance" from each of the candidates could also be used.

Hildreth[18] looked into the establishment of base alternatives as a way of choosing a unique utility function out of the class of those that are linear transformations of each other. He postulated that there exist two social states, X' and X'' such that each individual's prospects were the same in each state, and X' P_j X'' for all j where j refers to the citizen-voter-consumer. This implies that each individual's utility is identical in each of these states i.e. $u_j(X') = a$ and $u_j(X'') = b$. (a,b, constants, a > b, all j). These conditions do result in the unique specification of one utility or preference function for each individual. However, there are two problems with this approach. 1) It is somewhat artificial to suppose that there is universal agreement concerning any two social states and that the utilities for all individuals are equal under either of these two social states. 2) The inputs to the system from each individual are still not *normalized* in the sense that each have the same weight. X' and X'' are not necessarily the most preferred and

least preferred states for any individual so that the maximum and minimum utilities for each individual could be anything thus violating the democratic condition of equal weighting of inputs. If X' and X'' were postulated to be the states of maximum and minimum utility, respectively, for each individual, then objection 2 could be gotten around at the expense of intensifying objection 1. However, if X' and X'' are taken to be not objective states but the ideal best and worst states *for each individual voter*, then we have the approach taken in this paper.

The problem with choosing ideals and placing them in the option set is that every voter is going to want to maximize the effectiveness of his vote. Therefore, he will apply a linear transformation to his preference or presumed utility data so that his most preferred actual alternative will be placed at 1 and his least preferred actual alternative will be placed at 0. This would also be the result as we have seen above if the SDF does the normalizing. Therefore, it seems reasonable to modify Arrow's requirement concerning the independence of irrelevant alternatives rather than give up the democratic principle of equal weighting of inputs.

Returning to Arrow's example again

$P_1(x) = P_2(x) = 1, \quad P_3(x) = .5$

$P_1(y) = P_2(y) = .9, \quad P_3(y) = 1$

$P_1(z) = P_2(z) = P_3(z) = 0$

If z is considered the negative ideal, the elimination of z would not change the other ratings since z would still be included in the option set although it could not be chosen as an actual option. If we proceed under the assumption that each voter's input should carry equal weight and z is eliminated as an actual choice, then the change in outcome is a rational and logical consequence of the requirement that each vote should carry equal weight. In other words, if there are changes in the option set, the inputs must be renormalized, and this might effect changes in the outcome. This is a rational and non-arbitrary result.

CONDITIONS TO BE IMPOSED ON SOCIAL WELFARE FUNCTION

We wish to impose conditions as similar as possible to Arrow's original conditions in order to show that it was his insistence on pairwise comparisons that was primarily responsible for his impossibility result. If anything we will strengthen the conditions to be imposed.

1. Unrestricted Domain. Among the options in the option set, S, any individual citizen can assign any real-valued utility to any option i.e. $U_j(X) = y$ where $0 \leq y \leq 1$, for all options and all individuals.

This strengthens Arrow's Condition 1 which only requires a "free triple."

2. Positive Association of Social and Individual Values. If any individual increases his individual utility for any option, all other utility data for all other individuals remaining constant,

then that option must either rise or remain the same in the social rating. For some j and X, say j* and X*, $U'_{j*}(X^*) > U_{j*}(X^*)$. For all other j and X, $U'_j(X) = U_j(X)$. Then $C'(X^*) \geq C(X^*)$.

3. Independence of irrelevant alternatives. If an alternative is added to or deleted from an option set and this alternative is not a pole for any individual, i, then the social choice function will be identical in the two cases. In the case where the addition or deletion represents a pole for some individuals, the utility function of those individuals will change after the addition or deletion, and the social choice function may change.

Let X* be an alternative added to or deleted from an option set. Then for all j such that $U_j(X^*) > U_j(X)$ or $U_j(X^*) < U_j(X)$ for all X, $U'_j(S) \neq U_j(S)$. For all j for which the condition doesn't hold, $U'_j(S) = U_j(S)$ where $U'_j(S)$ represents the utilities before the change and $U_j(S)$, after.

4. Citizen's Sovereignty. The social welfare function should not be imposed i.e. for some pair of distinct alternatives, X and Y, X R Y for any set of individual orderings $U_1,...,U_T$, where R is the social ordering corresponding to $U_1,...,U_T$.

5. Nondictatorship. The social welfare function shall not be dictatorial i.e. there is no individual, j*, such that, for all X and Y, X P_{j*} Y implies that X P Y regardless of the orderings, $U_1,...,U_T$, of all individuals other than j*.

In addition we propose the Condition of Equal Option Sets. The option sets for all individuals are to be identical.

PROOFS

Condition 1 needs no proof. Proof of Condition 2 is as follows: Let us assume a maximization of social utility social decision function. Then we have for each option, $U(X) = \sum_{j=1}^{N} U_j(X)$. C(S) is a real-valued function of

U(X). For some j* and X*, let $U'_{j*}(X^*) > U_{j*}(X^*)$ and $U'_j(X) = U_j(X)$ for all $j \neq j^*, X \neq X^*$. Then $U'(X^*) > U(X^*)$ and $U'(X) = U(X)$ for all $X \neq X^*$. Therefore, $C'(X^*) > C(X)$.

Proof of Condition 3. Let X* represent an alternative added to or deleted from the option set. By assumption, for all j, X* is not a pole. Therefore, $U'_j(X) = U_j(X)$ for all j and all $X \neq X^*$ where U' represents the utilities after the change since each option's utility is independent of every other option's except for the two poles.

Proof of Condition 4. Assume X P Y for any set of utilities U(S). Now

$U_X = \sum_{j=1}^{N} U_j(X)$ and $U_Y = \sum_{j=1}^{N} U_j(Y)$. There is a set of utilities such that

$U_j(Y) > U_j(X)$ for all j. Therefore, $U_Y > U_X$ and $C(Y) > C(X)$ according to the maximization of social utility social decision function which contradicts the assumption.

Proof of Condition 5. Assume there is a j* such that X P Y regardless of the utilities, $U_1,...,U_T$. Consider a utility distribution such that $U_Y > U_X$. Then $C(Y) > C(X)$ and Y P X which contradicts the assumption.

OTHER CRITICISMS OF CARDINAL UTILITARIANISM

The main way that social choice theorists have chosen to get around Arrow's objection to the feasibility of interpersonal comparisons is through the use of extended sympathy.[19] It is argued that each citizen can put himself in each other citizen's shoes and express his preferences regarding the overall social state as if he were at that person's station and in that person's situation. We reject this particular approach to interpersonal comparisons for the following reasons. 1) It would be impossible for each citizen to identify with each other citizen's station or situation from the point of view of the sheer massivenessness of the data with which

he would have to become familiar. 2) It would be possible for some citizens to express extended antipathy rather than extended sympathy by either stating preferences which maximized their own welfare and minimized that of others or identified some individual or class for whom their stated extended preference was the *opposite* of the stated preferences of that individual or class. 3) We reject the notion of a "moral guardian"[20] who will make decisions for society because of his advanced moral wisdom or expertise. Part of the liberal agenda is to protect individuals from the arbitrary decisions of human agents either in or out of government. This is why the impersonal mechanism of voting with power dispersed among the people is preferable to decision making by a benign dictator or philosopher king. Placing social decision making processes in the hands of a moral guardian is a reversion away from democracy towards oligopoly or dictatorship.

For similar reasons to the above, we reject the idea that the individual in his statement of preferences in the economic sphere should order social states rather than his own individual work-consumption preferences. The individual's specification of preferences should not give him the power to affect the individual work-consumption schedule of some other individual. To state this more formally, for all j, if X P_j Y implies X P Y, there is no person or group of persons, j*, such that Y P_{j*} X for this person or group implies that Y P X where X and Y are alternatives that apply only to individual j. Each person should submit his preferences regarding his own work-consumption state and then the social decision function should, according to its inherent values, decide on the distribution of individual states. It should be pointed out that an individual can still express his tastes and values. He can be egoistic or altruistic in his own preferences. It's just that he cannot earmark some other particular individual or group to either help or hurt. In political decision making where the purpose of the process is to determine a social state rather than a combination of individually tailored solutions, this problem doesn't arise. Arrow states that, "...purely individualistic assumptions are useless in analyzing such problems as the division of the national income between public and private expenditure."[21] This is not true. An individual can include publicly consumed items

as well as privately consumed items on his preference list along with his preferences regarding how much of a contribution he is willing to make in order to pay for them. Thus taxation would not necessarily be distributed equally among the population but would be implicit in the maximization of social utility if in fact that were the social decision rule. Also items which could be consumed either publicly or privately could be included in the preference schedule both ways and the decision made about public or private consumption would be implicit. For instance, a person might prefer to have a pool in his back yard but might prefer a neighborhood community pool even more if the cost to him, personally, was sufficiently less.

We are concerned mainly in this paper with only two social decision processes: the political process such as the selection of a President, a legislative body or a direct vote on issues and the economic as in the selection of a social state which involves the specification of individual work-consumption schedules. The economic system we visualize is driven by the individual preference specification process much as democratic decision making is driven by the voting process in that public policy is not set by an expert, be he a welfare economist or a moral guardian, but by an impersonal algorithm responding to the aggregate of individual preference schedules. Just as voting represents an expressed rather than a revealed preference, we assume that individual economic preferences are expressed rather than revealed and that an individual's "vote" reveals what his presumed utilities would be as a function of actual outcome. As such we envision an application of political science to economics in the reverse of what Anthony Downs did in his book, "An Economic Theory of Democracy."[22] Our work concerns a political theory of economic democracy.

The argument about 'desire-based' or 'choice-based' utility is moot. An individual can express a preference or a set of preferences that is either egoistic or altruistic i.e. his preference schedule can be based completely on self-interest or completely on helping others or anywhere in between. It is the individual's right to decide how egoistic or how altruistic he will be in the specification of his preference schedule. Clearly, if a social or individual decision process results in an individual's being allocated a certain result corresponding to which he

has previously expressed a certain utility which we will take to be identical to his preference rating, then that individual will be more delighted the higher the utility is regardless of what is the mix between egoism and altruism that that result represents. In other words the result may represent satisfaction of personal desires to some extent and a voluntary giving to others to some extent. The individual's happiness is not strictly a function of his desire satisfaction but is also a function of his opportunities for altruism. Therefore, his utility contains components of both these needs. It is easy to see that a lower utility might contain more desire satisfaction and less altruistic satisfaction, and a higher utility, less desire satisfaction and more altruistic satisfaction. There is no reason why a unidimensional expression of preference identical to presumed utility cannot incorporate both. Indeed, in order to keep individual inputs to the decision mechanism equal, each individual's preference specification *must* incorporate both to a degree decided on by the individual. Once the individual outcomes have been allocated by the SDF, if an individual's actual, experienced utility is not equal to his presumed utility as specified at the input to the process, then the individual will have learned something regarding his utilities and will be able to come up with a preference or presumed utility schedule which more closely reflects his true utilities on the next go round. A learning process will have occurred!

Some social choice theorists object to the use of cardinal preference information on practical grounds. They maintain that it is impossible for an individual to accurately specify his preference information. However, there are expedient ways to do this at least in some cases. For instance, let us assume we are electing a legislature and the problem is to specify cardinal preferences over all those running. For those who are incumbents we can examine their voting records and compare them to how we would have voted. On a scale from 0 to 1, a legislator who voted as we would have 60% of the time would be rated .6 etc. For non-incumbents, they could each specify how they would have voted and can be judged accordingly. Finally, all this could be done automatically using a computer that contained the legislators' voting records in

memory. All that would be required would be to input the way the voter would have voted on the issues.

One of the problems in social choice theory has to do with the idea of unlimited domain. Some people seem to think that every logically possible option must be included in the alternative set over which the voters specify their preferences.[23] Thus we have writers using examples involving torture, sadism etc.[24] It would seem that inappropriate options would be excluded in the first place according to some qualification rules. For instance, in electing a legislature, the alternatives would be human beings who met certain requirements as to age, citizenship etc. Also it would be unthinkable to have as a legitimate alternative "Congressman A is to be elected under the condition that he is to be tortured by Congressman B." If such were the case we might want to elect the Congressman we hated the most just so we could have the satisfaction of having him tortured! The ruling out of inappropriate alternatives does not in any way restrict the logical placement on preference schedules of the acceptable alternatives.

Much attention in the literature has also been given to the notion of protecting individual rights within the social decision making process.[25,26,27] It has been shown that this is a difficult if not impossible thing to do. I would maintain that individual rights should be protected *outside* the decision process. If those rights are non-negotiable, they shouldn't even be a part of the process. For instance, the American Bill of Rights is something that stands outside the voting process. What is voted on whether in terms of candidates or issues is what is in some sense "up for grabs." Therefore, what are considered rights should never be a part of the voting process. In the economic arena, certain economic rights such as a minimum standard of living should be placed outside the arena of social decision making.

PART 2

CHOOSING A SOCIAL DECISION FUNCTION

Whereas Arrow's original work showed that there were no social decision functions that satisfied certain criteria, after the problem has been restated, the opposite problem occurs: there are many social decision functions which meet our standards - an embarrassment of riches. Which of all of these is best? The two most likely candidates presented in the literature are the Benthamite summation of utilities and the Rawlsian[28] maximin. In the Benthamite SDF, utilities are summed over all individuals and all alternatives. The alternative with the highest social utility wins, followed by the alternative with the second highest etc. In the Rawlsian SDF that alternative is chosen which results in the maximization of the minimum individual utility over all individuals.

In addition to there being many possible SDFs, there is the possibility that many solutions will satisfy any particular SDF. For instance, in the Benthamite maximization of utilities, there might be many solutions all of which produced the same maximum value. The question arises: could we apply additional ethical constraints in order to achieve a unique solution? Let us start with the Benthamite SDF. After we have maximized utility, which of the many resulting solutions do we pick? Let us assume that our goal now is to maximize equity. Of all the solutions which maximized social utility, we would pick those which maximized the minimum utility. Then assuming that there is more than one of these, we would pick those that minimized the number of people at this minimum utility level. Then we would proceed to the next lowest utility level, and of the remaining solutions we would pick the ones that maximized this next lowest utility level. Then of those remaining we would pick the ones that minimized the number of people at this level. Then we would proceed to the next lowest utility level and so on until we had gone through all the utility levels (assuming a finite number of them) and had picked those solutions at each level that first maximized the utility at that level and

secondly minimized the number of people at that level. After doing all of this we still might not have a unique solution. There is still the possibility that individuals i and j, for instance, are interchangable between levels A and B i.e. either i has to end up at A and j at B or j has to end up at A and i at B where A and B are utility levels and we can assume A > B. In order to resolve these cases, it should be possible to work out a situation in which the individual at the higher utility level pays a compensation to the one at the lower utility level resulting in an equalization of the two utility levels. When this has been done for all individuals in this kind of situation, a unique solution will have been arrived at.

Another possible solution would be to first apply the Rawlsian maximin criterion followed by a minimization of the number of individuals at the lowest utility level. Then of the remaining solutions find the ones that maximize utility. Then maximize the second lowest utility level and minimize the number of people at that level. Continue upward etc. Various combinations of the Benthamite SDF and the Rawlsian SDF could be tried. The pure Rawlsian approach would be to continue up through all levels maximizing utility and minimizing the number of individuals at that utility from the lowest level to the highest.

A third appraoch would be to set the minimum utility level and then maximize overall utility subject to that condition. This level would have to be set less than the maximin level. After utility had been maximized subject to this condition, the maximin criterion could be applied by levels starting with the next to lowest.

On the microcosmic level, let us assume we are considering moving to social state B from social state A, and that all utilities are the same in each state except for two individuals, i and j. Let us assume i has utility p in A and utility q in B, and j has utility r in A and s in B and p > r. If $q \geq p$ and $s \geq r$, then clearly B should be chosen over A. This corresponds to Pareto optimality. B is clearly the state that would be chosen by both the Benthamite and the Rawlsian criteria since the social utility of B is greater than that of A and the minimum utility of the two individuals in question is greater in B than in A. If $q \leq p$ and $s \leq r$, then clearly $U(B) < U(A)$ and the state A should be chosen for the same reasons. If $q \geq p$ and $s \leq r$ and $U(B) > U(A)$, then

the Benthamite criterion would choose B and the Rawlsian criterion would choose A. If $q \geq p$ and $s \leq r$ and $U(B) < U(A)$, then A would be chosen according to both criteria. If $q \leq p$ and $s \geq r$ and $U(B) > U(A)$, then B would be chosen by both criteria. If $q \leq p$ and $s \geq r$ and $U(B) < U(A)$, then B would be chosen by the Rawlsian criterion and A would be chosen by the Benthamite criterion.

GENERAL IMPLICATIONS

We[29] take a different approach than Riley[30] in the way that we resolve the shortcomings of purely Benthamite utilitarianism. There are two issues here: interpersonal comparisons of utility and ethical issues. Riley seeks to resolve these issues by making utility multi-dimensional. He postulates one set of utilities that are purely egoistic in nature and then some other sets that are altruistic and aesthetic in nature. In addition he assumes that there is at least one "moral guardian" who is a person who is capable of accurately identifying each individual's true utilities by means of the process of extended sympathy. When these non-equal utilities are fed into a purely Benthamite SDF, the issue of multiple social choices or indifferences at the social choice level is dealt with by going to a different dimension of utility and recomputing the social choice. Continuing in this way through the different dimensions of utility, the final social choice can hopefully be made. Our criticisms of this approach are the following:

1) the necessity for a "moral guardian" implies an ad hominem solution to the problem. It reintroduces the need for a government of men and not laws. It thus contradicts one of the basic tenets of liberalism which is the protection of the individual from personal power whether within the government or without it. The danger is that the moral guardian(s) can become immoral, that the philosopher king can become a dictator, that the oligarchy of morally superior people can become elitist.

2) the ethical inadequacy of the purely Benthamite SDF is resolved by resorting to obtaining other sets of more altruistic utilities from the population. The problem is that there is nothing to prevent any individual from representing his strictly egoistic utilities as altruistic utilities if he is so inclined. So the set of altruistic utilities could be a masquerade for egoistic utilities to some extent.

3) the necessity of collecting several sets of utilities places additional burdens on the citizenry and complicates and enlarges the societal computation process thus making the scheme less workable.

We have chosen the following alternative approach:

1) Protect the individual from human vagaries in government by postulating a completely impersonal SDF mechanism into which is built the ethical principles of society.

2) Demand that input preference or utility specifications be equal in their voting power. This is how the problem of interpersonal comparisons is resolved - as it is in traditional democratic voting systems.

3) Allow the individual complete freedom in the specification of his utility or preference input. He is free to be totally egoistic, totally altruistic or somewhere in between since there is no way to prevent him from being one way or the other no matter how many dimensions of utility are introduced.

4) The question of individual rights both poltitical and economic is settled outside of the social decision process *a priori* and is not subject to the voting process. Minimum levels of economic well-being as well as privacy and other political, civil and human rights are assumed to be *by definition* outside the purview of social decision making.

5) Equality of inputs to the system implies equality of voters' option sets or equal liberty.[31]

6) Protection of the individual from ad hominem interference in the social decision process implies that voter-worker-consumers do not have decision making power over the social state as it directly affects some other individual or group of individuals other

than society as a whole i.e. if there are a number of individually tailored solutions rather than one overall social solution, no individual or sub-group should have decision making power as to the solution affecting some other individual or sub-group. Each individual's input should be a specification as to his egoistic and altruistic demands on the system as a whole and the SDF then determines the final state.

POLITICAL IMPLICATIONS

The main difference between political and economic applications of SDF theory is that in the political world the social choice represents typically one alternative that applies to all voters as in the election of a President. Even in the election of a legislature which involves the election of a number of individuals, the exact composition represents one alternative among many. In the economic world we are dealing with multiple social choice solutions, ideally one for each voter-worker-consumer. In other words the overall social solution is composed of a number of individually tailored individual solutions equal to the number of citizens. Between these extremes it is possible to visualize political-economic systems in which the number of social solutions is greater than one but less than a number equal to the number of citizens in the society. As this number increases the system becomes relatively more economic, and , as this number decreases, the system becomes relatively more political. We will consider two political applications: the election of a President and the election of a legislature.

In the election of a President or in the selection of any other alternative which applies universally to every member of society, the Benthamite SDF would seem to be more appropriate than the Rawlsian. The general problem is the selection of one alternative out of N alternatives where N is an arbitrary integer greater than one. Maximization of social utility would be the direct extension of majority rule. It is conceivable that the application of the Benthamite SDF would result in a multiplicity of results which then could be narrowed down to one by the application of the Rawlsian techniques outlined in the last section. A multiplicity of

possible results, however, would be less likely than in the economic world. There would be less justification for using a purely Rawlsian SDF primarily because of the centralization of results: one result applying to all. In general as the decentralization of results increases (as the results become more individually tailored), there would be less justification in applying a purely Benthamite SDF. The use of the Benthamite SDF with full cardinal and ordinal information from each voter would result in the following improvements over strict majority rule:

1) All candidates could be voted on simultaneously instead of narrowed down to two in primary elections or by the use of run-offs. Theoretically, this is a much sounder approach resulting in greater voter satisfaction;

2) Voter preference information involving lower order (other than first) preferences is utilized in the selection process. This makes it more likely that there will be a consensus.

The rating of each candidate by the voters could be simplified by the voter's first determining where he stands on the issues. The candidates would then be required to state where they stood on the issues. A preference rating could then automatically be determined by each voter for each candidate by comparing the voter's position with each of the candidates' positions. This process could be automated. In general the candidate(s) who completely agreed with the voter's positions would be given the highest rating (a one), and the candidate(s) who disagreed with the voter on every issue would be given the lowest rating (a zero). Most candidates would fall somewhere in between. Note that there may not be any candidates at either 0 or 1 for any particular voter. In addition to their stands on the issues, the candidates could be rated for competence, effectiveness, ability to get the job done and general character. These ratings could also be factored in. It would be assumed that political experts of all persuasions would offer their recommendations and make them available to the voters in order to expedite the voter's job.

In general in a step by step process involving primaries or run-offs and only first preference votings, information is lost along the way resulting in less than maximum voter satisfaction. In

a one step procedure involving complete voter preference information over the complete domain of alternatives, the most satisfactory social choice will be made.

In the election of a legislature involving K members, the problem is to choose K members out of L candidates where L > K. This problem would be identical to the Presidential problem if each voter specified his preferences over all the possible combinations of L people taken K at a time. However, the process can be simplified if we assume that a voter's utility over an K member legislature can be decomposed into the sum of his utilities over each of the members. Then the voter's job is just to express his preferences over all the individual candidates, a much easier job, and then the social choice can be made by summing each candidate's utility over all the voters and picking the K candidates with the K highest social utilities. The overall social utility then is the summation of utilities over the K elected candidates. The assignment of utilities by each voter can be greatly simplified by the following procedure. As above, the voter would determine his positions on a finite number of issues. These could include issues that have come up over the last period of incumbancy for which the incumbents' voting records would be available as well as issues likely to come up over the next incumbancy period for which both incumbents and challengers would have to specify their positions. Challengers would also have to specify how they *would* have voted over the issues that the incumbents had voted upon. Of course allowance would have to be made for the fact that hindsight places one in a more perspicacious position. The legislators' records on initiating legislation and in general overall effectiveness could also be factored in. The result is that the voter would only have to vote once - on the issues - and the rest could be computed automatically.

The SDF could be set up such that there would be a certain specified minimum utility that would be guaranteed i.e. the process would culminate in such a way that everyone would have at least that much utility. This utility would be somewhere between 0 and the Rawlsian maximin value, and would place the SDF somewhere between purely Benthamite and purely Rawlsian. This would be the generalization of the concept of proportional representation in which every minority above a certain minimum percentage of the population is guaranteed

representation proportional to its size or number of votes. In the present case the size of the smallest minority that could be accomodated would be reduced to one person or to the individual level.

ECONOMIC IMPLICATIONS

We assume an economy where each individual's option set comprises a set of work-consumption states. Each individual specifies his preferences over this set, utilities are normalized and then become inputs to the SDF. The SDF then aggregates and integrates the input data to come up with the overall social choice which represents an aggregate of individually tailored solutions subject to the restraints of availability of resources and that the total work required be just sufficient to produce the total consumption required. The economy is driven by individual worker-consumer demand, and the output states are arrived at impersonally by the SDF mechanism. Since each input is given equal weighting, the economy is democratic, demand-driven and individually responsive.

The option sets, which in some ultimate sense are the same for all, are for practical purposes individually tailored in the work aspects by the fact that not all individuals are qualified for all jobs. An individual's qualifications would affect which jobs would be available to him. For instance, society would not let an individual practise medicine unless he had graduated from medical school. All consumption states would be available to all individuals. An individual's preference set might not be the same as his option set for the following reason. There may be options available to him which he may want to rule out as choosable options by the SDF. For instance, although qualified to practise medicine, an individual might decide that under no circumstances did he wish to practise medicine but instead wanted to work in another field for which he was also qualified. In our formulation, society cannot coerce the individual to perform a productive or consumptive function which he does not choose to include in his preference set.

The individual work states in the preference set consist of the amounts, types and distribution over time of work that the individual is prepared to do. The individual consumption states consist of quantities, qualities and types of goods and services which the individual chooses to consume including those goods and services which are consumed collectively and are considered to be public rather than private resources. Each work-consumption state would consist of a work state and a corresponding consumption state, and the individual would express his ordinal and cardinal preferences over each state which is acceptable to him.

The base states, those corresponding to utilities of 0 and 1, may be limited by society. For instance, society might place a ceiling on the number of hours any individual would be allowed to work in a week as well as a ceiling on the consumption of goods and services. These maximal states would not have to be included in an individual's preference set, but he could not exceed them. Likewise a floor could be placed under the allowable minimal states for each individual in terms both of hours worked and goods consumed. For instance, it could be decided that each individual, at a minimum, would be required to work 10 hours per week and would receive goods and services amounting to $10,000 a year, roughly the poverty level. This would be the state corresponding to the base level 0. This would also be the default state - the state that would be chosen for any particular individual by the SDF if his more preferred states were not acceptable to society as determined by the SDF criteria.

In such an economy, prices could be rationally set. For instance, the price of a particular product could be set equivalent to the number of hours of labor that went into producing it. Note that hourly wages would vary in accordance with the final disposition of individual economic states decided upon by the SDF. The wages for each individual would correspond to an amount of money just sufficient to purchase the items corresponding to his consumption state. Thus an individual's hourly wage need not correspond to the amount of money required to purchase a product that was produced with one hour of labor.

The individual consumption states could either be specified explicitly or inferred from polling data which would allow the individual then to specify his consumption states in terms of wages or salary. A typical work-consumption utility schedule is shown in Figure 7.

FIGURE 7 Here

Notice that the worker-consumer's ideal state is a work week of 40 hours at a salary of $1200. This represents an hourly wage of $30. He is willing to work "overtime" with a 50 hour week at $40/hour or a 60 hour week at $50/hour although he attaches 0 utility to the latter state. He is not willing to work more than 60 hours/week since no work states higher than 60 hours/week appear on his schedule. He is willing to work less than 40 hours a week at the rate of $30/hour though at diminishing utility, and, finally, we have the default state of 10 hours/week at a salary of $200.

The law of supply and demand would be subsumed in such an economy. First supply would equal demand since it would be the responsibility of the SDF to allocate labor in such a way as to produce just what was demanded by consumers. However, let's assume that there is an undersupply of labor to meet the demand for the items produced by that type of labor. What would happen is that the available labor would be asked to work overtime to produce more and that consumers would be asked to consume less. Just which laborers would be asked to produce more and how much and just which consumers would be asked to consume less and how much would be determined by the SDF acting with the knowledge of the resource constraints involved and acting to produce an equilibrium and some ethically constrained version of maximum satisfaction. In other words consumer dissatisfaction at not having more would be balanced by worker dissatisfaction at working more. New workers could be transferred and/or retrained in order to alleviate the burden on the "overemployed" workers.

Let us assume that we have an oversupply of some type of labor or an oversupply of some type of good. Then workers will be asked to work less and consumers to consume more until

there is a balance as determined by the SDF. The underemployed workers can be transferred to another part of the labor force where they are more needed and/or retrained if the situation is not just temporary.

Taxation in such an economy would be to some extent a voluntary matter. For some worker-consumers, their work-consumption schedules may contain an altruistic component i.e. they may knowingly choose to work more and/or consume less in order to contribute something to society. This would be in effect a self-imposed and voluntary form of taxation. Society's needs for public consumption are part of the individual's preference schedule so both the amount and the funding of public or societal consumption is implicit in the process of choosing the final social state by the SDF. In effect the public would vote on the kinds and amounts of public consumption and the SDF would balance public consumption with the public's willingness to pay for it. Taxation would not be uniform but would be implicit in the social decision process. Society's fixed needs i.e. those that were not subject to being voted upon such as the base-line economic allocations would also come out of the SDF process presumably lowering overall social utility by some amount. This would all be handled in an optimal way by the SDF itself.

Private property could be incorporated into the above process making it applicable not only to socialist economic systems but capitalist and mixed systems as well. For instance, instead of an individual specifying his work preferences as inputs, he might specify instead a set of goods and services he will supply in return for the set he will consume. In general he might have a set of goods, services and labor as inputs (his consumables) and a set of goods, services and labor as outputs (his work).

Of course he would have to state a whole set of such states with his preferences over them. This then would be his input to the system and the SDF would decide which of his possible states would be his actual or final state. Also a person might be the owner of some natural resource so that his input to the system would be a set of states in which a certain amount of

this resource was offerred (replacing labor) in return for goods and services. Again the SDF would determine the final state.

EMBODYING ETHICAL VALUES IN SDF

Finally, we look at the structure of the SDF itself to determine which ethical values to embody in it. Let us look at the possible transactions that could take place on the microcosmic level to see whether or not they are ethical. We postulate two social states A and B and two individuals i and j with work-consumption states (W_i,C_i) and (W_j,C_j), respectively. Let p and r be the utilities of i and j, respectively, in state A, and q and s be the utilities of i and j, respectively, in state B. Let $(W_i,C_i)^A$ and $(W_j,C_j)^A$ be the work-consumption states of i and j in state A, and $(W_i,C_i)^B$ and $(W_j,C_j)^B$ be the work-consumption states of i and j in state B. Let us assume that all individuals other than i and j have the same social utilities in states A and B, and the social utility of state B is greater than the social utility of state A. Also p >r. Now if q > p and s > r both the Benthamite and the Rawlsian SDFs would choose state B over state A by Pareto optimality. If q < p and s < r, both SDFs would also choose B providing q > r. If q < r, we would have a case similar to the one considered below. If q < p and s < r, this would imply U(B) < U(A) which is impossible by assumption. The only case left then is q > p and s < r. In this case the Benthamite SDF would choose B and the Rawlsian SDF would choose A. There are 16 sub-cases to consider based upon the quantities $(W_i,C_i)^A$, $(W_j,C_j)^A$ and $(W_i,C_i)^B$, $(W_j,C_j)^B$. In general both work and consumption in state B can be either less than or greater than work and consumption in state A for both i and j. The logical possibilities are shown in Figure 8. where < means that the quantity is less in state B than in state A etc.

FIGURE 8 Here

Cases 3, 7 and 15 represent transfers from individual j to i. In each case j works more and consumes less in B than in A and his utility declines from A to B. Individual i, therefore, is better off in B at j's expense. In cases 2, 10 and 14, there is a transfer from i to j although j is worse off after the transfer. We would call this "imposed altruism," a misguided kind of altruism that makes the recipient worse rather than better off. The first situation we call "imposed egoism" or exploitation. From an ethical point of view, we reject both of these situations. Note that cases 6 and 11 are impossible because of the balance between supply and demand. Cases 5 and 8 also imply imposed egoism but not as strongly as the other cases. Finally, we are left with 6 cases, 1, 4, 9, 12, 13 and 16 in which there is no clear-cut imposed altruism or egoism. We would choose to apply the Benthamite criterion in the latter 6 cases and the Rawlsian criterion in the other 8 excluding the two impossible cases. Thus we are taking a middle road between the protective morality of the Rawlsian criterion and the maximization of utility of the Benthamite criterion. The corresponding composite SDF would produce a solution which combines aspects of the other two SDFs.

Once the composite SDF has been applied, the multiple solutions, if there are any, can be reduced by the methods discussed earlier.

SUMMARY AND CONCLUSIONS

We take issue with Arrow's assumptions on three grounds:

1) the issue of ordinal versus cardinal information;

2) the issue of interpersonal comparisons;

3) the issue of irrelevant alternatives.

We show that interpersonal comparisons can be handled on an objective basis providing that individual preference schedules are normalized and used as equally weighted inputs to the SDF. We compute a measure of both the ordinal and cardinal information contained in a preference schedule. This information is invariant under linear transformation of the preference schedule. We introduce a new perspective on the issue of irrelevant alternatives

and show that the principle of equal weighting of inputs and the establishment of base alternatives means that the principle of independence of irrelevant alternatives needs to be modified.

Democracy implies that individual preference schedules be given equal weight as inputs to the SDF both in political and economic systems. Also the option sets themselves must be identical for all citizens so that each individual has equal freedom of choice. Liberalism has sought first to protect individuals from government and, secondarily, to protect individuals from other individuals. We utilize these principles in rejecting the idea of a "moral guardian" whose purpose is to decide the legitimate differences in weightings of preference or utility schedules by means of "extended sympathy." We also reject the notion of multi-dimensional utilities since the relevant information can be expressed in one dimension and there is no way to control the relative egoism or altruism of the individual in his expression of preferences or presumed personal utilities. In the economic model in which there are individually-tailored social solutions we reject the notion that individual citizens should express preferences over other citizens' individual economic states. This would allow individuals to affect other individuals adversely as well as beneficially in the decision process itself. The protection from this possibility lies in the fact that the ethical values of society are manifested and invested in the SDF itself and not necessarily in individual utilities and in the fact that the SDF is an impersonal mechanism. Thus individuals are protected from people in government and from private citizens as well.

Individual rights are handled outside the social decision making process. This resolves the problems involved in trying to include them in the social decision process. According to Rawls, "...[T]he rights secured by justice are not subject to political bargaining or to the calculus of social interests."[32] Accordingly we proscribe the inclusion of inappropriate options in option sets such as those involving torture etc. We also reserve the right for the individual not to include a logically possible option in his preference schedule if it is one which he doesn't want chosen by the SDF under any circumstances. Those options which are included, however, can be expressed in any form without restriction.

Additional ethical criteria can be used to reduce multiple social solutions to a singular solution. For instance, if a Benthamite SDF is used initially, Rawlsian criteria can be used to inject equity and select out the ultimate social choice. Finally, the ethics of the SDF itself are examined. We consider the political models of election of a President and election of a legislature. The political model is exemplified by a social decision problem in which the social decision applies to all (no individual tailoring). When there is one social solution which applies to all, a minimum utility can be set somewhere between 0 and the Rawlsian maximin, and then social utility maximized subject to that restraint. This would be a generalization of the concept of proportional representation.

In the economic model we consider which involves an application of political theory to economics, the economy is demand driven by worker-consumer preference schedule inputs weighted equally. The social choice represents an aggregate of solutions individually tailored rather than one solution applied to all. The social choice mechanism is impersonal and embodies the ethical value judgments of society. We examine the microcosmic ethics involved in choosing between different social states and how individuals are affected in different situations. We suggest that certain situations call for Benthamite ethics and certain, Rawlsian ethics so that the overall SDF becomes a composite. The ethics involved in the overall social choice as manifested by this SDF contain components of both Benthamism and Rawlsianism.

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