Utilitarian Social Choice With a Maximin Provision

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Abstract

Utilitarianism has been faulted for maximizing utility or well being without regard to the least well off. For example, in a utilitarian society the societal well being might be maximized by neglecting the well being of a small number of individuals who have a rare disease that is very expensive to treat. It is proven herein that the Optimal Threshold Social Choice system produces the utilitarian winner which maximizes total social utility. This system then can include a measurable and calculable maximin provision that raises the utilities of the least advantaged individuals at the expense of a decrease in overall social utility. This can be done systematically so that the maximin condition can be accomplished while diminishing the social utility by the least amount. An application of these principles to the selection of a representative body is considered. It is shown that the processing power available today in computer chips used for artificial intelligence can also be used to expand democracy by eliminating districting so that an individual voter can vote on selecting members over all seats of a representative body.

Introduction

In the debate among principles of distributive ethics, two of the main contenders are contractualism as exemplified by John Rawls (2001), and utilitarianism as represented by John Harsanyi (1977), Amarya Sen (2002), Hun Chung (2023) and others. T.M. Scanlon (1982) writes, "Contractualism has been proposed as the alternative to utilitarianism before, notably by John Rawls in *A Theory of Justice*." While these writers consider an all encompassing view of social morality, our interest here is restricted to a particular system of social choice, the Optimal Threshold Social Choice (OTSC) system (Lawrence, 2023). We abstract from issues of personal morality and assume that all available choices within the system are morally acceptable to society in general. As Harsanyi (1977)points out: "we must exclude all clearly antisocial preferences, such as sadism, envy, resentment and malice." Arrow (1951) writes, "In the theory of consumer's choice each alternative would be a commodity bundle; ... in welfare economics, each alternative would be a distribution of commodities and labor requirements. … in the theory of elections, the alternatives are candidates." Clearly, Arrovian social choice abstracts from antisocial preferences. In a cash economy, however, which includes most modern economies, a "distribution of commodities and labor requirements" might be replaced by "a distribution of compensation levels versus labor requirements" with each individual specifying their utilities over a range of options. In this paper we consider only the theory of elections although the results can be applied to the other arenas that Arrow and others have suggested.

Kenneth Arrow's book, *Social Choice and Individual Values*, (1951) purportedly proved that social choice was impossible. Individual preferences couldn't be amalgamated into a social preference in such a way as to meet certain rational and normative conditions. Also, Gibbard (1973) and Satterthwaite (1975) proved that it was impossible to

amalgamate individual preferences in such a way that there was no advantage to any individual to use strategy to get a better result for themselves. We have shown (Lawrence, 2023) that with a utility based system, rather than the preference based system that Arrow assumed, Arrow's criteria can be met and actually surpassed.
Lehtinan (2015: p.35) has pointed out that the use of strategy tends to increase social utility: "strategic behavior increases the frequency with which the *utilitarian winner* is chosen compared to sincere behavior ". The utilitarian winner is the one that maximizes social utility. This is crucial in the sense that, unless a system that maximizes social utility is available, it makes less sense to suggest a maximin provision in which the individual utilities of the least advantaged are raised at the expense of a diminution of total social utility. It also is important to be able to calculate how much social utility is lost in order to obtain the maximin provision. In this paper we prove that the OTSC system produces the utilitarian winner(s). Furthermore, we show that the OTSC system can incorporate the Rawlsian maximin or "difference principle" in a measurable way which elevates the utilities of the least well off at the expense of a systematic and measurable diminution of overall social utility. We provide the outline of a computer program to do so using basic logical commands.

Ari Berman (2024) pointed out that democracy in the United States was limited by the US Constitution of 1789 because of a number of provisions: 1) the fact that black slaves were to be considered 3/5 of a person in order to increase the apportionment of seats in the House of Representatives for southern slave holding states; 2) the fact that each state was given two senators regardless of that state's population giving disproportionate power to states with the least population; 3) gerrymandering which allowed the party that controlled state legislatures to draw the boundaries for districts in such a way as to favor that party; and finally 4) the electoral college which gives the most power for electing the US President to a few battleground states. However, Berman doesn't consider the reduction in an individual's voting power brought about by the de facto system of districting itself.

A national congress or assembly should represent all the people. In the US Congress, representatives are elected district by district. In the House there is one congressman from every district. They serve their constituents in that district primarily and secondarily the nation at large. Similarly, in the Senate there are two senators from each state who serve the interests of their constituents in that state. So each American votes for only three national representatives – one congressman and two senators – and is primarily represented by those representatives. A true national congress would be one in which all representatives were voted upon and selected by all citizens. If all citizens get to vote for all representatives, the Congress would be truly districtless. This would result in a veritable expansion of democracy The political parties might evaluate the candidates and make recommendations so that the individual voter would not have to evaluate each candidate personally. For instance, if the Democratic party recommended an entire slate of candidates, and a voter wanted to vote a straight Democratic ticket, she would just put a "D" next to each candidate that she wanted to be elected presumably in an online ballot. In this paper we do the calculations that show that a districtless Congress is possible with today's computer technology. This would advance the cause of direct democracy in which every representaive would represent every voter.

Social Choice History

Although we analyze a particular system of utilitarian voting or choosing, the OTSC system, the analysis could also proceed just by assuming that each individual has a set of utilities and that the winner(s) of the election is the one or ones that result in the highest summation over those utilities. This would be similar to range or score voting (Smith, 2023) in which each candidate is assigned a number from 0 to 10, for instance. Then the scores for each candidate are simply the sums over all voters. In the system we consider here, each candidate similarly is assigned a utility from 0 to 1. However, range or score voting as well as utilitarian voting (Hillinger, 2005) does not pass Arrow's Impossibility theorem. The OTSC system does. We will proceed with the OTSC system because it provides for a more exact and precise understanding. The OTSC system results in the selection of a winning set of representatives from among the number of candidates under the voters' consideration.

The Arrow and Gibbard-Satterthwaite impossibility theorems were based on the following representation of individual preferences: $aR_jb \ldots R_jz$ where a, *b, ..., z* are alternatives, *j* represents an individual voter/chooser and R_j means preferred or indifferent to. The social choice then is expressed as $aRb \ldots Rz$. We may refer to this as a preference based method.

Arrow (1951: p. 32) considered but rejected the possibility of using a utility based method. After proposing to measure utilities on a scale from zero to one, he says: "It is not hard to see that the suggested assignment of utilities is extremely unsatisfactory. Suppose that there are altogether three alternatives and three individuals. Let two of the individuals have utility 1 for alternative x, .9 for y, and 0 for z; and let the third individual have the utility 1 for y, .5 for x and 0 or z. According to the above [summation of utilities] criterion, y is preferred to x. Clearly, z is a very undesirable alternative since each individual regards it as worst. If z were blotted out of existence, it should not make any difference to the final outcome; yet under the proposed rule for assigning utilities to alternatives, doing so would cause the first two individuals to have utility 1 for x and utility 0 for y, while the third individual has utility 0 for x and 1 for y, so that the ordering by sum of utilities would cause x to be preferred to y."

Arrow fundamentally misunderstands the assignment of utilities. Consider the following example. The scale again consists of all real numbers between 0 and 1, but there need not be a candidate identified as having a utility of 0 or 1. The scale is independent of the actual assignment of utilities. Let individual 1 assign alternative x to utility 0, alternative y to 0.1 and alternative z to 1. Now consider alternative z to be "blotted out of existence." Individual 1 should still rate alternative y as 0.1 for the following reason. Let's say that alternative y is a SOB in individual 1's opinion. Just because alternative z is "blotted out of existence" doesn't mean that individual 1 has changed his opinion of alternative y and should assign him a utility of 1. Alternative y is still a SOB in individual 1's opinion and should still be assigned a utility of 0.1. In Arrow's framework, alternative y would be assigned a utility of 1. This is clearly ridiculous. That would be elevating form over function!

Hillinger (2004, p. 3) has also made the case that utilitarian style sincere ratings for each candidate are assumed to be independent of each other regardless of the composition of the alternative set. "A cardinal number assigned to an object indicates its place on a scale that is independent of other objects." Also see Lawrence (2023, p. 21): "A candidate's dropping out or entering the race is assumed not to change an individual's sincere ratings for the other candidates."

Arrow (1951: p. 10-11) has a problem with the comparability of individual utility indicators. “Even if, for some reason, we should admit the measurability of utility for an individual, there still remains the question of aggregating the individual utilities. At best, it is contended that, for an individual, their utility function is uniquely determined up to a linear transformation; we must still choose one out of the infinite family of indicators to represent the individual, and the values of the aggregate (say a sum) are dependent on how the choice is made for each individual. In general, there seems to be no method intrinsic to utility measurement which will make the choice compatible.”

For the OTSC system choosers can place their respective utilities for alternatives on a scale of their own choosing on a line consisting of the set of all non-negative real numbers, $\mathbb{R}^+$, and also choose the end points. Crucially, and contrary to Arrow's suggestion, the values of the aggregate are not a sum! In the OTSC system any affine linear transformation of an individual's set of utility ratings will yield the same output or social choice results, and, therefore, it doesn't matter which scale an individual chooses. Rather than summing individual utilities, the OTSC system does a unique transformation *for each voter* from their cardinal inputs to their AV (approval voting) style contribution to the social choice output. The system processes the inputs in such a way as to maximize the expected utility of the social choice for each individual chooser based on their choices alone. Therefore, the individual has no incentive to "cheat" or use strategy. Assuming no knowledge of the statistics of other individual utilities or polling data, it turns out that the best strategy is to place an optimal threshold in each individual's input data and to give each alternative above threshold a utility rating of "1" and each alternative below threshold a utility rating of "0." The number of above and below threshold candidates will be the same regardless of the scale chosen, and therefore, an affine linear transformation can be applied to each individual input before it is processed by the OTSC system so that their utilitarian ratings for the candidates are expressed on a scale with "0" and "1" as the end points. Utilitarian style inputs are converted to approval style outputs by the OTSC system. Each individual's input is converted to a set of votes for the alternatives or candidates. Therefore, the issue of interpersonal comparisons is moot because, regardless of the scale chosen by each individual, the number of candidates above and below threshold will be the same for that individual. The votes are then tallied to determine the winner(s).

The OTSC System

Let $C = \{c_1, c_2, ..., c_n\}$ be an ordered set of candidates/alternatives of size n; candidates appear on the ballot in $c_1, c_2, ..., c_n$ order. $V = \{v_1, v_2, ... v_q\}$ is a set of voters or choosers of size q, where $v_j \in V$ denotes the j^{th} voter/chooser. The set of voters can also be considered to be ordered, for instance, alphabetically. Each individual submits a two part ballot ordering the candidates in order of their own preferences, $C_j = \{c_{1j}, c_{2j}, ... c_{nj}\}$ and also a concomitant set of utiities, $U_j = \{u_{1j}, u_{2j}, ... u_{nj}\}$. U_j is the utility set of the j^{th} voter after applying an affine linear transformation to their submitted set of utilities so that $0 \leq u_{ij} \leq 1$. u_{ij} is the utility of candidate c_{ij}. The OTSC system puts a unique and optimal threshold into the set of utilties for *each voter/chooser* which turns the set of utilities into a set of approval style votes.The optimal threshold is the one that maximizes the expected utility of the winning set for each individual voter.
$B_j = \{b_{1j}, b_{2j}, ..., ,b_{nj}\}$ is the set of approval style votes in order of the j^{th} voter's candidate preferences. $b_{ij} = \{ \mathbb{N}^0 \mid 0, 1\}$. The votes are then tallied for each candidate. The winning set, $W = \{w_1, w_2, ..., w_m\}$, represents an unordered set of the m candidates with the highest number of votes. However, it can also be considered an ordered set by virtue of the number of votes obtained by each winning candidate. The utility of the winning set for each individual, j, can be calculated since *j's* utility for each member of the winning set is known. *j's* utility for the winning set then is the sum of *j's* utilities for each member of the winning set. This can be normalized by dividing by the number of candidates in the winning set to get individual *j's* utility per representative. The social utility is the sum over the utilities of each individual voter/chooser. The fact that each individual voter/chooser's utility for the winning set can be computed makes it possible that the winning set can be altered in such a way as to raise the utilities of those with the least individual utilities. Thus a measurable maximin condition can be effectuated.

The OTSC system actually satisfies all of Arrow's "rational and normative" conditions, and is even normatively more robust than Arrow demanded because the utilitarian data is more finely tuned than preference data. Instead of the individual preference profiles that Arrow assumed for individual j - aR_jb ... R_jz - a is preferred or equivalent to b etc., we assume individual utilitarian inputs in which the alternatives, after a linear transformation, have corresponding utilities between zero and one. The result is that each individual's self determined scale of utilities is converted to a set of utilities on the scale from "0" to "`1". Then in order to forestall the individuals' use of strategy to change their sincere utilities, we let the OTSC system itself apply the optimal strategy for each individual thereby dissuading individuals from doing so. Subsequently, after the insertion of the optimal threshold into each individual's utility set, the utilitarian style inputs are converted to approval style outputs. Therefore, the OTSC system is a utilitarian, approval (UAV) hybrid system.

The OTSC System Picks the Utilitarian Winner

The utilitarian winner is the one that maximizes the social utility of the social choice. The OTSC system actually picks the utilitarian winner. We prove this as follows with reference to the terminology of *Proving Social Choice Possible.*(Lawrence, 2023)

Consider the winning set, $W = \{w_1, w_2, ..., w_m\}$, which consists of the m candidates who received the highest number of votes. The set, $Y = \{y_1, y_2, ..., y_n\}$, orders the candidates by the number of votes received by each candidate. $y_1Ry_2R ... Ry_n$. Let ${}^{A}u_j$ be the utility of the winning set, W, for individual voter/chooser j post-election, and ${}^{A}u$ be the social utility of the winning set for all voter/choosers - the utility of the social choice.

$$^{A}\boldsymbol{u}_j = \sum_{i=1}^{m} \boldsymbol{\eta}_j \boldsymbol{\varkappa}_j \boldsymbol{\tau}^{-1} \boldsymbol{\alpha}^{-1} \boldsymbol{\beta}^{-1} (\boldsymbol{w}_i)$$

where η, χ, τ, α, and β are defined as follows:

i) $\beta : Y \rightarrow W$ such that $\beta(\mathrm{y_i}) = w_i$ for $1 \leq i \leq m$. The function, β, places the top m vote getters in the winning set. If y_m represents a tie with y_{m+z} for $z \geq 1$, ties are resolved randomly so that W is always of size m.

ii) $\alpha : X \rightarrow Y$ α defines an ordered pair (x_r, y_r) such that [$y_r \mathrm{R} y_z$ iff $x_r \geq x_z$]

for $1 \leq r, z \leq n$; r, z, n *integers.*

iii) $\tau : C \rightarrow X$ defines an ordered pair, (c_i, x_i) such that $\tau(c_i) = x_i$,the cumulative number of votes for each candidate.

iv) $\chi_j : C \rightarrow C_j$ The function χ_j assigns to each element $c_i \in C$ an element $\chi_j(c_i) = c_{ij}$ such that $c_{1j} \mathrm{R_j} c_{2j} \ldots c_{(n-1)j} \mathrm{R_j} c_{nj}$ for $1 \leq j \leq q$ where $\mathrm{R_j}$ means "is preferred or indifferent to". Each voter, j, orders the set of alternatives according to their preferences. There are q voters.

v) $\eta_j : C_j \rightarrow U_j$ the function η_j assigns to each element $c_{ij} \in C_j$ an element $\eta_j(c_{ij}) = u_{ij}$ where u_{ij} is the utility that is assigned to candidate c_{ij} by voter j.

The social utility of the winning set is

$$^{\mathbf{A}}\mathbf{u} = \sum_{\mathbf{j}=1}^{\mathbf{q}} {}^{\mathbf{A}}\mathbf{u}_{\mathbf{j}}$$

Proof by contradiction:

Consider the candidate, $y_m = \beta^{-1}(w_m)$, in the winning set who has the least number of votes, let's say x_m votes. Discounting ties, the next highest ranked candidate, y_{m+1} has at most x_m *-1* votes. Replace y_m in the winning set with y_{m+1}. Call this set *W'*. Assume that the set *W'* has greater total utility than the set *W*. Therefore,

$$^{A}\mathbf{u}_j = \sum_{i=1}^{m} \eta_j \chi_j \tau^{-1} \alpha^{-1} \mathbf{y}_{m+1} > \sum_{I=1}^{m} \eta_j \chi_j \tau^{-1} \alpha^{-1} \mathbf{y}_m$$

and

$$^{A}\mathbf{u}_j = \eta_j \chi_j \tau^{-1} \alpha^{-1} \mathbf{y}_{m+1} > \eta_j \chi_j \tau^{-1} \alpha^{-1} \mathbf{y}_m$$

But by assumption, $x_{m+1} = \alpha^{-1}(y_{m+1}) < x_m = \alpha^{-1}(y_m)$

Therefore, the utility of the set, *W'*, is less than the utility of the set, *W*, and the winning set, *W*, represents the set with the highest total utility, the utilitarian winner.

A Maximin Strategy for the OTSC System

After the optimal set has been established and the output utilities computed for each individual, the worst off person or set of persons in terms of utility might have their results improved at the expense of a diminution of total social utility. One way to do this is as follows. Starting with the worst off individual or set of individuals, make all possible changes to the winning set and calculate the worst off set's utility and also the total utility after each change has been made. If the worst off set's utility can be increased in such a way that the total utility is not decreased more than that increase, then this would be a possible maximin solution.

An algorithm for this process would be as follows. Consider the entire set of candidates: $C = \{c_1, c_2, ..., c_n\}$. Now consider every permutation of this set taken *m* at a time as a potential winning set, $W = \{w_1, w_2, ..., w_m\}$, for that permutation. Let's call this a provisional winning set and compute the individual and total social utilities for this set. For each permutation, if there is an improvement of the utilities of the set with the lowest utility in such a way that the diminution of total utility is not greater than this improvement, then this is a possible maximin solution. The best maximin solution would be the one such that the set with the lowest utilities is improved the most while diminishing total social utility the least.

There are alternative ways of coming up with a maximin solution. One could determine a minimum utility level for all individuals and then compute the best way to achieve this which would result in the least diminution of social utility. Again all permutations of the candidate set could be considered to determine which winning set accomplishes this.

The Outline of a Programming Solution

Appendix A contains the outline of a computer program which computes the winning set of candidates and also a minimax solution. For example, we assume 200 million voters which is larger than the number of registered voters in the US, a winning set of 500 which is larger than the number of people in the US House of Representatives and 1000 candidates. Then we compute the amount of time necessary to implement this program with the assumed parameters to see if a districtless House of Representatives is realistic with today's technology using the characteristics of an advanced computer chip used for artificial intelligence. Our model for the maximin solution is to determine the winning set such that everyone has at least the minimum utility for this winning set of representatives if possible in such a way as to diminish total social utility the least. If this is not possible, we determine that solution with the minimum number of individuals below the acceptable minimum level of utility.

This program continues until a representative winning set is found such that no voter has a minimum utility less than 0.1. However there may be no such set in which case we calculate the representative set with the least number of voters with a minimum utility less than 0.1. This might be the best we can do. These calculations are possible because we have a complete ordering over all the candidates in terms of the number of votes received. Also we can calculate the individual and social utilities for any potential set of winning candidates. Therefore, we can degrade the set of winning representatives systematically and either pick that set such that there is no voter with a utility less than the minimum acceptable utility, or such that, if no such set exists, there is a minimum number of voters with a utility less than the minimum acceptable utility.

Computing the Number of Calculations

We add up the number of calculations required to find the winning representative set with the following assumptions:

numvot = 200 million **an integer representing the number of voters

numcan = 1000 **an integer representing the number of candidates

numrep = 500 ** an integer representing the number of representatives

1) Summing collective utility for each candidate over all voters: numcan x numvot additions e.g. $2 \times 10^8 \times 10^3 = 2 \times 10^{11}$ calculations, approximately.

2) For minimax condition: numrep x numcan x numvot additions to compute individual utilities for rep set: e.g.. $5\times10^2 \times 10^3 \times 2 \times 10^8 = 10^{14}$ calculations, approximately.

Computing the individual and social utilities to determine a maximin solution should be no problem at all with the computing power available today. According to The Verge, "Nvidia says the new B200 GPU offers up to 20 *petaflops* of FP4 horsepower from its 208 billion transistors." Petaflop is a unit of computing speed equal to one thousand million million (10^{15}) floating point operations per second. FP4 means four bits of floating point precision per operation. 10^{15} is the same as 20,000 trillion or 20 quadrillion. Considering the fact that there are about 170 million registered voters in the US and the US House of Representatives has 435 members, it would be within the realm of possibilty to elect directly the entire House using the computing power available today. In fact the Nvidia B200 GPU could do all the calculations in a couple of seconds!

Summary and Conclusions

It has been determined that the Optimal Threshold Social Choice (OTSC) system overcomes Kenneth Arrow's Impossibility Theorem circa 1951. OTSC is a utilitarian approval (UAV) hybrid system in which individual utilitarian style inputs are converted to binary votes for candidates/alternatives by using strategy to come up with the most advantageous way of doing so for each individual. Individuals are dissuaded from using the optimal strategy since the OTSC system does it for them. The output of the system represents the social choice consisting of the winning candidate or candidates. Since the underlying utilities of each individual voter/chooser are known, individual output utilities can be computed as well as the overall social utility.

It has been shown that the OTSC system produces the utilitarian winner(s) which is the winner(s) which maximizes social utility. There are a number of possible maximin solutions which would elevate the utilities of those individuals whose output utility results are the lowest. For instance, since both individual and collective utilities can be calculated for any set of winning candidates, the winning set can be degraded systematically to determine the set with the largest social utility that satisfies the condition that everyone has at least a minimum acceptable individual utility for the outcome of the election.

The outline of a computer program is presented in Appendix A which shows the algorithmic flow of commands to produce the utilitarian winner for a representative body such as the US House of Representatives. An algorithm for a minimax solution is also shown. The algorithm shows first the utilitarian solution which maximizes social utility. Then this algorithm considers all configurations of the winning set of representatives such that every voter has at least a minimum utility. If there is no such set, the algorithm finds that configuration of representatives such that the number of individuals with less than minimum utility is the least.

Rawlsian concern for maximin solutions has been integrated with the utilitarian concern for maximizing social utility. While Rawls' general arguments did not present any measurable or logical way of accomplishing this, we have by contrast presented a measurable, algorithmic solution. This solution for electing a districtless House of Representatives is possible using the computing power inherent in advanced computer chips available today which are mainly used for artificial intelligence.

Appendix A

i, j, k, m, p, q	** these variables are integer counters **i is an index which assigns an integer to all the candidates in alphanumerical order. 1 < i < 1000 e.g. if the candidates are Abbot, Costello and Barclay etc,, Abbot would be "1", Barclay, "2" and Costello, "3", etc. **j is an index which assigns an integer to all the voters in alphanumerical order. 1 < j < 200,000,000 e.g. if the voters are Adwell, Costner and Bergman etc., Adwell would be "1", Bergman, "2" and Costner, "3", etc. **m represents the configuration of the winning **representative set i.e. which candidates have been **elected **k, q and p are counters
numvot	**an integer equal to the number of voters
numvot = 200,000,000	**assumption for this example
numcan	**an integer equal to the number of candidates
numcan = 1000	**assumption for this example
numrep	**an integer equal to the number of representatives in the representative assembly
numrep = 500	** assumption for this example
numutil	**the number of possible utilities
numutil = 11	**assumption for this example: the set (0.0, 0.1, ..., 1.0).
util[i,j]	**an array which contains the utility of voter *j* for **candidate *i*
utilsum[j]	** an array which contains the total utility of the

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                                  **winning set of representatives for voter j
votcan[i, j)]                     **an array which contains the set of votes, B_j, for each
                                  **candidate e.g. votcan[12, 40] = 1 signifies that b_ij = 1
                                  **for the i = 12th candidate and the j = 40th voter .
canord [i]                        **candidate order after the voting process in terms of
                                  votes, B_j e.g. canord[1] = i would represent the fact that
                                  candidate i received the most votes.
votsum[i]                         **sum of votes for candidate i over all voters
utilmin                           **minimax condition. everyone should have at least this
                                  **much average utility per representative if possible
utilmin = 0.1                     **assumption for this example
socutil                           **sum of utilities over all representatives and voters, the
                                  **social utility
kord                              **integer used in ordering the candidates

main program
for (i=1, numcan)
        votsum[i] = 0       **initializes votsum
end i
for (i=1, numcan)                 **sums votes for each candidate over all voters
        for (j=1, numvot)
                votsum[i] = votcan[i,j] + votsum[i]
        end j
end i
for  (i=1, numcan)                **orders the candidates
        kord = 0
        votsum[kord] = 0
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        for (k=1, numcan)
                if (votsum[k] > votsum[kord]
                        then
                                kord = k
        end k
        canord[i] = kord
end i

socutil = 0                                 **computes maximum social utility
for (i=1, numrep)
        for (j = 1, numvot)
        socutil = socutil + utilsum[canord[i]],j]
        end j
end i
```

Minimax Solution

**Program could stop here having computed set of representatives resulting in maximum utility. Program continues to compute social utility with minimax provision that each voter has a minimum average utility of 0.1 = utilsum[j]/numrep per representative. Program proceeds by systematically replacing members of winning set with non members. We don't impose the condition that the gain in social utility by those with utilmin < 0.1 should be no greater than the loss of total social utility

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flag                                        **an integer representing number of individuals
                                            **with utility for winning representative set less
                                            **than utilmin for a particular configuration of the
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                                        **winning set
flagmin                                 ** an integer representing the minimum number of
                                        **individuals with utilities less than utilmin over
                                        **all configurations of the winning set

canordtemp[k]                           ** an array that holds the current winning set of
                                        **representatives under consideration

canordmin[k]                            **configuration of representative set which results
                                        **in the minimum number of individuals with
                                        **utilities less than the minimum utility

for (p=1, numcan)
        canordtemp[p] = canord[p]       **this is the set that maximizes social utility
end p                                   **which will be systematically degraded
                                        **to find the set with the least number of
                                        **individuals with utilities less than utilmin
m = 0
flag = 0
a:      for (k=1, numcan)                       **replaces member of winning set with least
                                                **number of votes
                if(numrep + k + m) = numcan + 1
                        go to b                 **there are no more possible configurations
                        canordtemp[numrep] = canordtemp[numrep + k + m]

                for (j=1, numvot)
                        utilsum[j] = 0
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        for (i=1, numrep)
            utilsum[j] = utilsum[j] + util[canordtemp[i], j]
                                        **adds utilities of canordtemp set for voter j
                                        **computes voter j's total utility for a
                                        **particular set of  representatives
        end i
    if (utilsum[j]/numrep < utilmin)
        then
            flag = flag + 1             **count of voters with utilities less than
                                        **utilmin
        end j                           **go to next voter

if (flag = 0)                       **there are no individual utilities < .1
    then
    go to b                         **this configuration results in
                                    **every individual having at
    else                            **least  minimum utility
    for (i=1, numrep)
        for (j = 1, numvot)
        socutil = socutil + utilsum[[canordtemp[i]],j]
        end j
    end i

    if (flag ≤ flagmin)
                                        **flagmin represents minimum number of
                                        **voters with utilities less than minimum
                                        **utility. canordmin[k] is
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                                        **corresponding configuration of
                                        **winning set.

                flagmin = flag
                for (p=1, numrep)
                        canordmin[p] = canordtemp[p]
                end p
        end k

m = m + 1                               **set up next configuration
        for (q=1, m)                    ** moves non-winning candidates over one place
                                        **in winning set
                canordtemp[numrep+q-m] = canordtemp[numrep+q]
        end q
go to a                                 **go to computations for next configuration

b:      end  k                          **find final configuration of winning set

for (p=1, numrep)
        canord[p] = canordtmin[p]       **this is the final configuration such that as
end p                                   **few as possible individuals have
                                        **minimum utility
```

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